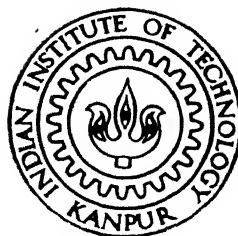


# AN INVESTIGATION INTO THE PROBLEMS RELATED TO THE DESIGN AND DEVELOPMENT OF RESEARCH STIRLING ENGINE

by

YOGESH KUMAR SINGH

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DEPARTMENT OF MECHANICAL ENGINEERING

**INDIAN INSTITUTE OF TECHNOLOGY KANPUR**

August, 1996

An Investigation into the Problems  
Related to  
the Design and Development  
of  
Research Stirling Engine

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for the Degree of  
MASTER OF TECHNOLOGY

by

Yogesh Kumar Singh

to the

Department of Mechanical Engineering  
INDIAN INSTITUTE OF TECHNOLOGY, KANPUR

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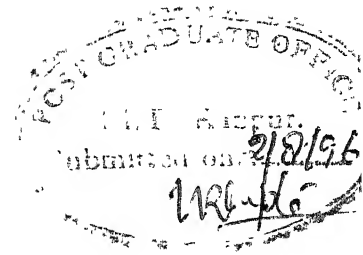
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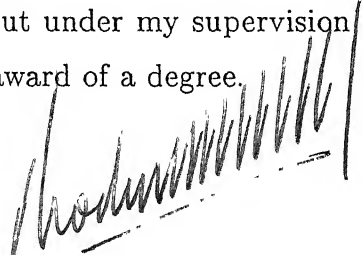
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# CERTIFICATE



It is to certify that the work contained in this thesis entitled "An investigation into the problems related to the design and development of a research Stirling Engine" by Yogesh Kumar Singh, has been carried out under my supervision and that this work has not been submitted elsewhere for the award of a degree.

  
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August, 1996

## ABSTRACT

The present work is directed towards the fabrication of a design for a Stirling engine which is suitable for purpose of further research and data generation on the subject. An existing design of Peter Meede was studied for same and a model was fabricated based upon it. A new design for a prototype Stirling engine was also indicated which stresses modularity and simpler sealing problems, which are the two major defects of the existing model. A  $\gamma$ - Stirling engine was finally chosen.

The modularity of the  $\gamma$ - Stirling engine will be of great help in changing Stirling engine parameters like temperature, pressure, swept volume, stroke, regenerator type as well as medium of heat transfer.

The research engine will serve the purpose of further work on this fascinating and still unexplored subject due to simplicity, multifuel capability, low pollution level, quiet operation and low torque variation.

# Dedicated to my Parents

## ACKNOWLEDGEMENTS

At the onset, I express my deep gratitude and sincere regards to my thesis supervisor, **Prof.P. K. Das** for his constant inspiration, encouragement and guidance throughout the work. His motivating guidance and constructive criticism were the main drives in completing this work.

I also take this opportunity to Thank the friends like Nilesh, Kale V.S., Gires, V. K. Srivastav, Ajay Das, K. D. Kumar, K. S. Yadav and to all my well wishers who made my stay at IIT Kanpur a memorable and had shared the great moments of joy and fun and helped me directly or indirectly whenever I needed it.

Yogesh Kumar Singh

# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
1.1	Invention of Stirling Engine . . . . .	1
1.1.1	Historical Background . . . . .	1
1.1.2	Other Contributors . . . . .	1
1.2	Working Cycle of Stirling Engine . . . . .	2
1.3	Characteristics and Applications of Stirling Engine . . . . .	4
1.3.1	Advantages . . . . .	4
1.3.2	Applications . . . . .	7
1.4	Scope of the Present Work . . . . .	8
1.5	Organization of the Thesis . . . . .	8
<b>2</b>	<b>Thermodynamic cycle of Stirling Engine</b>	<b>9</b>
2.1	Thermodynamic Cycle of Stirling Engine . . . . .	9
2.2	Comparison with Carnot Cycle . . . . .	12
2.3	Practical Regenerative Stirling Cycle . . . . .	12
2.3.1	Theoretical Assumptions . . . . .	12
2.3.2	Modifications in Practical Stirling Engine . . . . .	14
2.4	Mechanical Arrangements . . . . .	15
2.5	Control Systems . . . . .	15
2.6	Limitations . . . . .	18
<b>3</b>	<b>Stirling Engine Analysis</b>	<b>23</b>
3.1	Schmidt Analysis . . . . .	23



3.1.1	Mean Cycle Pressure . . . . .	26
3.1.2	Heat Transferred and Work Done . . . . .	26
3.1.3	Mass Distribution in the Machine . . . . .	28
3.2	Optimisation of Design Parameters . . . . .	29
3.2.1	Consolidated Charts . . . . .	30
3.3	Practical Stirling Engine Analysis . . . . .	30
3.3.1	C. D. West's Analysis of Stirling Engine . . . . .	32
<b>4</b>	<b>Fabrication and Analysis of <math>\beta</math> - Stirling Engine Model</b>	<b>35</b>
4.1	The Prototype Stirling Engine . . . . .	35
4.2	Details of the $\beta$ - Stirling Engine selected . . . . .	35
4.2.1	Technical Description . . . . .	35
4.2.2	Physical Dimensions . . . . .	36
4.2.3	Material Requirements of the Fabricated Model . . . . .	36
4.2.4	Power Estimation of Model . . . . .	36
4.3	Features of Engine Design . . . . .	51
4.3.1	Drawbacks . . . . .	51
4.3.2	Advantageous Points of Meede's Design . . . . .	53
4.4	Fabrication of the Model . . . . .	53
4.4.1	Manufacturing Difficulties . . . . .	53
4.4.2	Assembly Problems . . . . .	54
4.4.3	First Running Test . . . . .	54
4.4.4	Frictional Force Measurement . . . . .	55
<b>5</b>	<b>Investigation into <math>\gamma</math> - Stirling Engine</b>	<b>58</b>
5.1	Objectives of the New Design . . . . .	58
5.2	Visualization of Design . . . . .	59
5.3	$\gamma$ - Stirling Engine Parameters . . . . .	59
<b>6</b>	<b>Results and Conclusions</b>	<b>63</b>

## REFERENCES

# List of Figures

1.1	The basic Stirling engine. . . . .	3
1.2	The displacer and power piston and their movement in a Stirling engine. . . . .	4
1.3	The working cycle of Stirling engine and its indicator diagram. . . . .	5
2.1	(a) P-V and T-S diagram of Thermodynamic Stirling cycle. (b) Piston positions at the terminal points of the cycle (Walker, G. <i>et al</i> [4]). . . . .	10
2.2	Stirling versus Carnot cycle (Walker, G. <i>et al</i> [4]). . . . .	13
2.3	Different Stirling engine configurations : (a) $\beta$ - configuration, (b) $\gamma$ - configuration and (c) $\alpha$ - configuration (West, C.D. [5]). . . . .	16
2.4	Effect of pressure increase on the power output and efficiency of a Stirling engine (Walker, G. <i>et al</i> [4]). . . . .	17
2.5	Power output of Stirling engine as a function of the speed (Walker, G. <i>et al</i> [4]). . . . .	18
2.6	Effect of Stroke length variation on pressure ratio and power output in a Stirling engine (Walker, G. <i>et al</i> [4]). . . . .	19
2.7	Effect of dead volume on power output in a Stirling engine (Walker, G. <i>et al</i> [4]). . . . .	20
2.8	Effect on power output of a Stirling engine due to variation in phase angle $\alpha$ (Walker, G. <i>et al</i> [4]). . . . .	21
3.1	Consolidated charts for preliminary design (Walker, G. <i>et al</i> [3] . . . . .	31
4.1	Fabricated $\beta$ - Stirling engine model. . . . .	37
4.2	Engine base plate. . . . .	38

4.3	Flywheel. . . . .	39
4.4	Bearing fork. . . . .	40
4.5	Engine stand. . . . .	41
4.6	Crank disc and axle. . . . .	42
4.7	Power cylinder. . . . .	43
4.8	Power piston. . . . .	44
4.9	Displacer cylinder. . . . .	45
4.10	Displacer piston. . . . .	46
4.11	Rectangular cooler. . . . .	47
4.12	Circular cooler. . . . .	48
4.13	Connecting rod and connecting links. . . . .	49
4.14	Table. . . . .	50
4.15	Graph showing the plot of frictional power and brake power against swept volume, after point P brake power takes over frictional power and region beyond it gives feasible design. . . . .	57
5.1	Proposed $\gamma$ - Stirling engine. . . . .	60

# List of Tables

4.1	Operating parameters and physical dimensions of the fabricated $\beta$ - Stirling engine model. . . . .	36
4.2	Raw material requirements of the fabricated model of $\beta$ - Stirling engine.	51
4.3	Design and performance parameters of fabricated $\beta$ - Stirling engine. . .	52
4.4	Frictional force measurement experiment. . . . .	55
5.1	Design and performance parameters of proposed $\gamma$ -Stirling engine. . . .	61
5.2	Optimum design and performance parameters of proposed $\gamma$ -Stirling engine.	62

# Nomenclature

A	A factor $(\xi^2 + k^2 + 2\xi k \cos \alpha)^{1/2}$ .
B	A factor $(\xi + k + 2S)$ .
f	Speed of engine.
M	Total mass of working fluid.
p	Instantaneous cycle-pressure.
$p_{max}$	Maximum cycle pressure.
$p_{mean}$	Mean cycle pressure.
$p_{min}$	Minimum cycle pressure.
P	Engine output.
$P_{mass}$	$P/RT_c$ , Dimensionless power parameter based on the mass of working fluid.
$P_{max}$	$P/p_{max}V_T$ , Dimensionless power parameter based on the maximum cycle pressure and combined swept volume.
Q	Heat transferred to the working fluid in the expansion space
$Q_{mass}$	$Q/RT_c$ , Dimensionless heat transferred to working fluid based on the mass of working fluid.
$Q_{max}$	$Q/p_{max}V_T$ , Dimensionless heat transferred to working fluid based on the maximum cycle pressure.
R	Characteristic gas constant of the working fluid.
S	$2X\xi/(\xi + 1)$ , Reduced dead volume.

$T_C$	Temperature of the working fluid in the compression space.
$T_D$	Temperature of the working fluid in the dead space.
$T_E$	Temperature of the working fluid in the expansion space.
$T_m$	Mean Temperature of the working fluid when piston is mid of the stroke.
$V_C$	Swept volume in compression space.
$V_{E-}$	Swept volume in the expansion space.
$V_D$	Dead volume, total volume of heater, cooler, regenerator and ducts and ports.
$V_m$	Total volume( $V_C + V_D + V_E$ )
$V_o$	Swept volume of power piston.
$V_T$	$V_C + V_E$ , combined swept volume.
$X$	$V_D/V_E$ , dead volume ratio.
$\alpha$	Angle by which volume variations in the expansion space lead those in the compression space.
$\delta$	$A/B, \frac{(\xi^2 + k^2 + 2\xi k \cos \alpha)^{1/2}}{(\xi + k + 2S)}$ .
$\theta$	$\tan^{-1}(k \sin \alpha / (\xi + k \cos \alpha))$ .
$k$	$V_C/V_E$ , swept volume ratio.
$\xi$	$T_C/T_E$ , temperature ratio.
$\phi$	Crank angle.

**Note** Lower case suffixes indicate instantaneous values while upper case suffixes show constant values or maximum values.

# Chapter 1

## Introduction

A Stirling engine informally known as *hot air engine* is based on an idea first proposed by an Scottish minister Robert Stirling in 1816 A.D. It is a device which operates on a closed regenerative thermodynamic cycle, with cyclic compression and expansion of the working fluid at different temperature levels, and where the flow is controlled by volume changes, so that there is a net conversion of heat to work or vice-versa.

### 1.1 Invention of Stirling Engine

#### 1.1.1 Historical Background

It was long before the invention of the petrol and diesel engine and at a time when steam engine was coming on the scene, the Stirling engine was invented in 1816 A.D. by Robert Stirling. Superior power-to-weight characteristics of the steam engine led to the abandonment of the Stirling engine. However, Stirling engine remained a matter of academic interest during whole of the nineteenth century because of its unique feature that, if the ideal Stirling thermodynamic cycle were practically realizable, the efficiency of a frictionless Stirling engine would be the same as that of a Carnot engine i.e. the theoretically attainable maximum efficiency in any heat engine.

#### 1.1.2 Other Contributors

Not much work was done on Stirling engine till 1938, when N. V. Philips of Netherlands started research on it. High thermal efficiency of the Stirling engine coupled with the fact



that an almost endless varieties of mechanical implementations are possible, led Philips to apply the knowledge of fluid dynamics and with the development of heat and creep resistant materials he successfully developed a small gas refrigerating machine based on Stirling principle as a popular and cheap method of producing liquefied gases. With the invention of rhombic drive <sup>1</sup> in 1953, Stirling engine has developed to a competitive stage with diesel and petrol engines. A thermal efficiency of 40 percent coupled with a specific power of 82 KW per liter of swept volume, have already been obtained and with continued research it should be possible to increase them quite considerably.

In 1971 Philips company in collaboration with Ford Motors monitored the performance of a closed cycle, four-cylinder, double-acting swashplate <sup>2</sup> drive Stirling engine using hydrogen as the working fluid in 2500 Kg inertia weight, class-1975 Ford Torino vehicle. This Stirling engine-powered vehicle exhibited excellent fuel economy, extremely low emissions, low noise level and performance comparable to that of a pre-emission controlled Otto engine with good drivability.

Most notable contributions among independent efforts other than Philips is, perhaps, by William Beale, a Professor at Ohio University, Athens. Professor Beale studied the Stirling engine in early 1960 and concluded that, at least in theory, the customary kinematic linkage coupling, crankshaft, reciprocating and displacer pistons were unnecessary. Thus was born the free piston Stirling engine.

## 1.2 Working Cycle of Stirling Engine

The basic principle of the Stirling engine (refer Figure 1.1) is a simple one and easily explained. being no more than the tendency of a gas to expand with rise in pressure when heated. A fixed amount of gas is contained in a working volume consisting of at least one space maintained at a high temperature and another at a lower temperature. Some amount of the gas is transferred back and forth between hot and cold space by means of

---

<sup>1</sup>It consists of two crankshafts rotating in opposite direction. two pairs of identical connecting rod and one pair of yokes make the rhombus for good balancing.

<sup>2</sup>Swashplate is a disclike rotor mounted concentrically with , but at a skew angle to obtain phasing of the pistons by peripheral positioning of the piston rod contacts.

piston movements. Pressure rises when most of the gas is in the expansion or hot space and falls when gas is transferred to compression or cold space. **Displacer piston** is the piston responsible for transferring the gas while the piston moved by high pressure is called **power piston** (refer Figure 1.2) which increases the gas volume. When the gas is moved back into the cold region the pressure falls then the gas is compressed back to its original volume by reversing the movement of the power piston. Due to low pressure, the force on the power piston is less during its inward compression stroke as compared that during the outward expansion stroke and it results in a net amount of work.

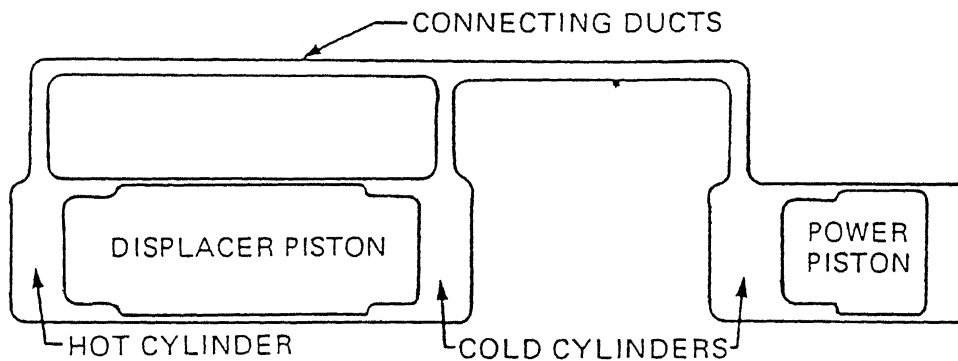


Figure 1.1: The basic Stirling engine (West, C.D. [5])

Figure 1.3 illustrates the entire cycle and the indicator diagram of ideal Stirling engine. The displacer is situated at the hot end of its cylinder, thus transferring whole amount of the gas to the cold end of the engine before beginning the compression stroke (refer Figure 1.3(a)). The compression stroke takes place compressing all of the working gas in the cold region of the engine (refer Figure 1.3(b)). After the compression stroke, the displacer is moved to the other end of the cylinder, which transfers the gas to the hot region, thereby raising the pressure in preparation for the expansion stroke (refer Figure 1.3(c)). At any given position of power piston, the pressure is higher on the outward stroke, when some of the gas is hot, than it was on the inward stroke, when

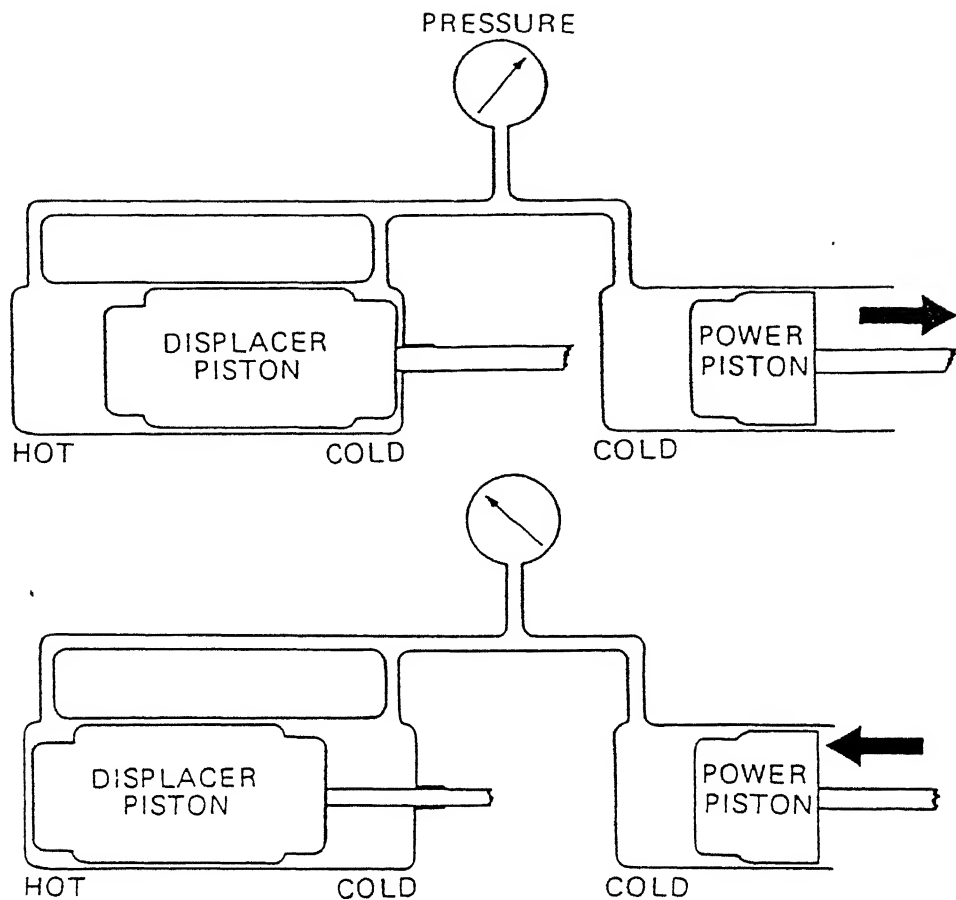


Figure 1.2: The displacer and power piston and their movement in a Stirling engine. (West, C.D. [5])

all of the gas is cold (refer Figure 1.3(d)). Therefore more work is done on the piston during the gas expansion than has to be done to recompress the gas, and the difference is the available work from the system. At the end of the expansion stroke, the displacer piston is returned to the hot end for the another cycle.

## 1.3 Characteristics and Applications of Stirling Engine

### 1.3.1 Advantages

Followings highlight the various advantages offered by Stirling engine in comparison with the other engines :

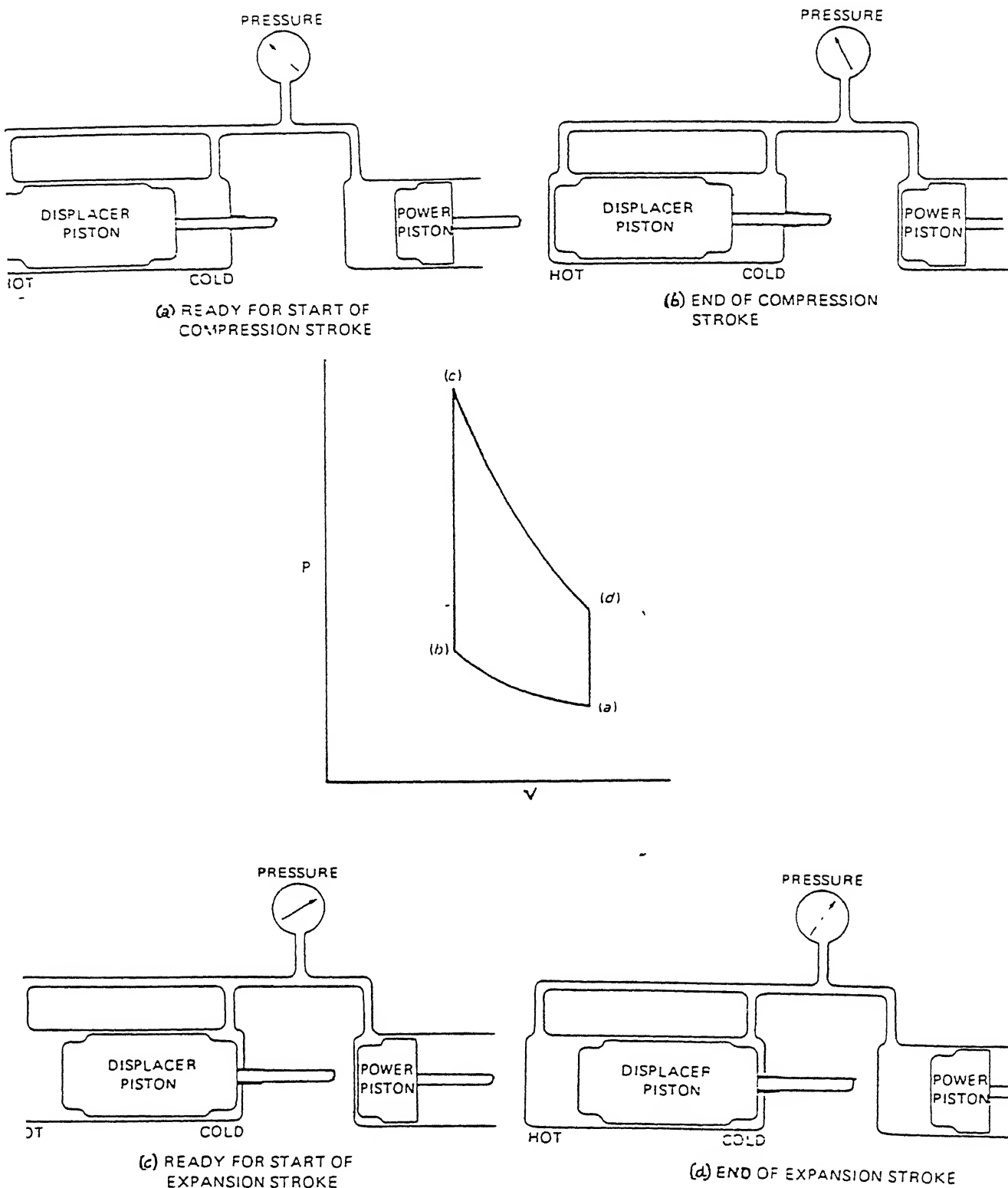


Figure 1.3: The working cycle of Stirling engine and its indicator diagram (West, C.D. [5])

1. Stirling engine is an **external combustion** engine. So, a wide variety of heat sources can be used. Also, Stirling engines have been tested practically to operate from solid, liquid, and gaseous fossil fuel combustion, solar heat, vegetable oil combustion, nuclear heating, heat stored in molten salts and combustion of biomass waste products. Furthermore, heat is supplied continuously during operation so that if a burner is the source of heat, the combustion process can be controlled and efficient. Consequently, the emission from a well designed Stirling engine are lower in comparison with an internal combustion engine and the adaptability to different fuels is much greater. The separation of the combustion processes from the internal working parts of the engine also avoids contamination and deterioration of the lubricants.
2. Since the **working fluid is retained in the engine**, rather than being ejected and replaced each cycle, it can be chosen on the basis of its properties and effectiveness. By a suitable choice of working fluid, efficiency-robbing processes such as flow losses can be minimized. hydrogen or helium is the usual choice, both gases combining low density and viscosity with good heat transfer properties. although air was used in the earlier engines and is still recommended for applications. such as farm equipment for developing countries, where low cost and ease of maintenance are the major design goals.
3. The **smooth pressure variation** that results from moving the working fluid continuously back and forth between hot and cold regions combined with the continuous combustion, as opposed to explosive combustion. result in a **quiet engine operation**, with a **very even torque** that is almost independent of engine speed.
4. Most of the heat rejected from the cycle is carried away by the coolant and is thus readily available for supplementary use, for example in a co-generation system. most of the heat rejected from the internal combustion engine is in the form of hot and corrosive exhaust gas and can be recovered only with difficulty.

5. The nature of the basic Stirling cycle with a regenerator is one that is thermodynamically efficient because heat is added only to gas, already at high temperature of the system and removed only from the gas, already at low temperature. A cycle with these isothermal processes has the highest theoretical efficiency permitted by the laws of Physics, i.e. Carnot efficiency. The efficiency of a practical Stirling engine is not, of course, so high, but the **freedom to choose the most appropriate working fluid** and the opportunity to incorporate a sophisticated combustor with exhaust heat recuperation means that the stirling engine can approach more closely than any other engine, at least in the medium and low power applications for which the large and complex systems of a central steam power plant are not practical.

### 1.3.2 Applications

The features that promote the particular characteristics of the Stirling engine make it uniquely appropriate for certain applications like :

- To power an implantable artificial heart due to its external heating capability,
- In underwater power units due to its silent operation for military submarines,
- In space power due to its ability to use nuclear heating, good efficiency,
- In military ground power due to low maintenance, capability of using any type of liquid fuels and quiet operation,
- In solar thermal conversion due to its ability to utilize the externally supplied heat at the best and good efficiency,
- As automotive engines due to multifuel capacity, stringent emission and low noise level,
- For heat pumping purposes as it is fossil fueled,
- Remote power sources.

## 1.4 Scope of the Present Work

A practical Stirling engine has numerous advantages described earlier. But, due to non-availability of a Stirling engine model the research on this attractive and much exploitable topic in India could not be carried out. So, the present work was undertaken aiming towards the investigation of the problems related to design and development of a Stirling engine model based on Peter Meede's design so as to provide the much needed product model for the future research on the unexploited subject of Stirling engine. It was also aimed to propose the design and thermodynamic analysis of a  $\gamma$  - Stirling engine prototype.

## 1.5 Organization of the Thesis

Chapter 2 describes the thermodynamic review of theoretical Stirling cycle and modifications required for its practical applications.

Chapter 3 presents thermodynamic analysis of Stirling engine.

Chapter 4 deals with fabrication and analysis of a  $\beta$  -Stirling engine prototype.

Chapter 5 proposes the new design of  $\gamma$ - Stirling engine.

Chapter 6 finally concludes with focusing on the possible future extensions.

## Chapter 2

# Thermodynamic cycle of Stirling Engine

### 2.1 Thermodynamic Cycle of Stirling Engine

Thermodynamic cycle of Stirling engine consists of two isothermal and two constant volume heat transfer processes (refer Figure 2.1(a)). Its working can be explained with reference to an engine containing two opposed pistons in a single cylinder with a regenerator between them (refer Figure 2.1(b)). This regenerator may be thought of as a thermodynamic sponge, alternately releasing and absorbing heat. It is a matrix of finely divided metal in the form of wires or strips. One of the two volumes between the regenerator and the pistons is called **expansion space** and it is maintained at a high temperature  $T_E$ . The other volume is called **compression space** and it is maintained at a low temperature  $T_C$ . There is a temperature gradient across the transverse faces of the regenerator and it is assumed that there is no thermal conduction in the longitudinal direction. It is also assumed that the pistons move without friction or leakage loss of the working fluid enclosed between them.

**PROCESS 1-2 :** The cycle starts with isothermal compression process in which working fluid is compressed and heat is transferred from the working fluid at  $T_C$  to the external sink to maintain the isothermal condition. At the start of the cycle the compression space piston is at the outer dead point and the expansion space piston is at the inner dead point, close to the face of the regenerator. All the working fluid is in the



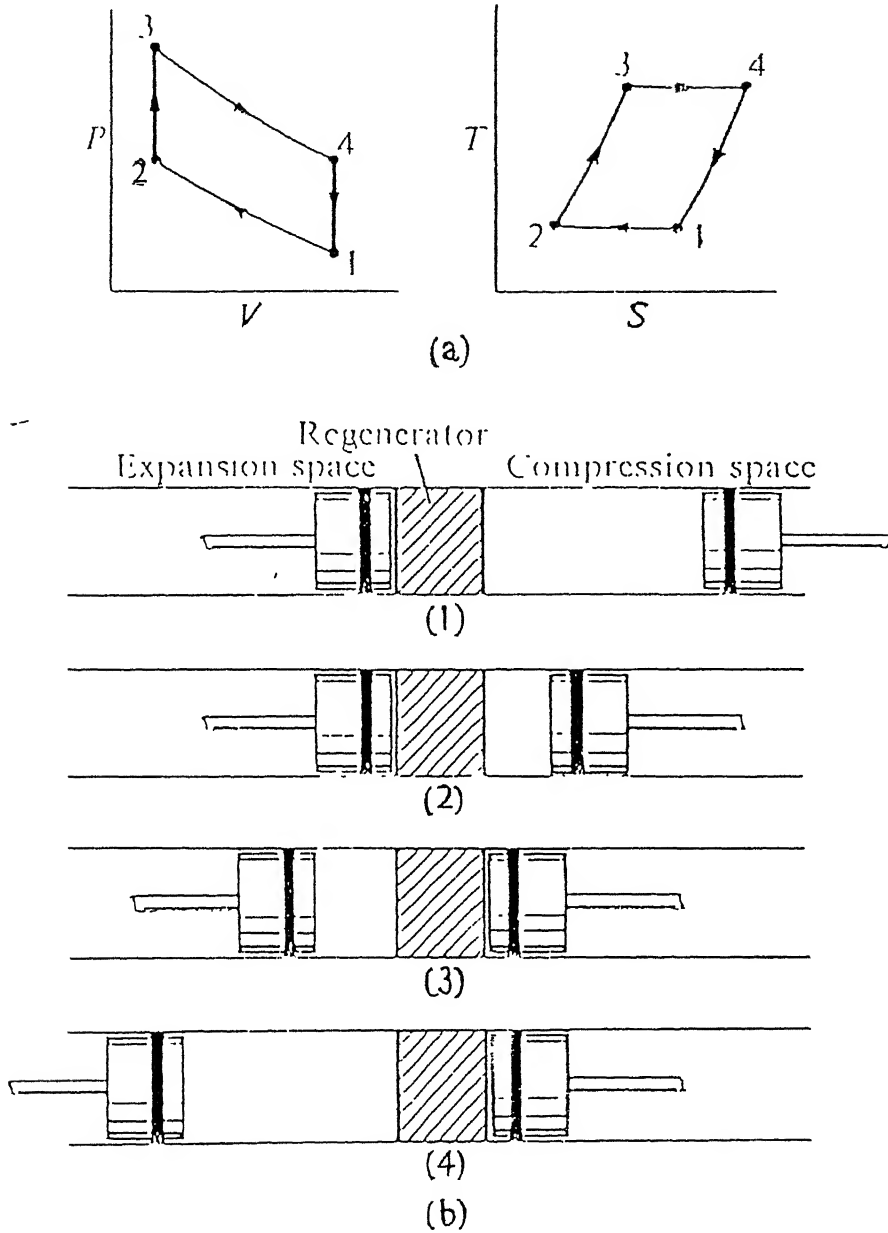


Figure 2.1: (a)  $P$ - $V$  and  $T$ - $S$  diagram of Thermodynamic Stirling cycle. (b) Piston positions at the terminal points of the cycle (Walker, G. *et al* [4]).

cold compression space. The volume is maximum, so that the pressure and temperature are at their minimum values, represented by point 1 on the P-V and T-S diagrams, shown in (Figure 2.1). During compression the compression piston moves towards the inner dead point and the expansion space piston remains stationary. The working fluid is compressed in the compression space and the pressure increases. The temperature is maintained constant because heat  $Q_c$  is abstracted from the compression space cylinder to the surroundings.

**PROCESS 2-3 :** is a constant volume heat transfer to the working fluid from the regenerative matrix. In this process both pistons move simultaneously, the compression piston towards (and the expansion piston away from) the regenerator, so that the volume between them remains constant. Therefore, the working fluid is transferred through the porous metallic matrix of the regenerator from the compression space to the expansion space. In passage through the regenerator the working fluid is heated from  $T_C$  to  $T_E$  by heat transfer from the matrix and emerges from the regenerator into the expansion space at temperature  $T_E$ . The gradual increase in temperature in passage through the matrix, at constant volume, causes an increase in pressure.

**PROCESS 3-4 :** is an isothermal expansion in which heat is transferred to the working fluid at  $T_E$  from an external source. In this expansion process the expansion piston continues to move away from the regenerator towards the outer dead point while the compression piston remain stationary at the inner dead point, adjacent to the regenerator. As the expansion proceeds, the pressure decreases as the volume increases. The temperature remains constant because heat  $Q$  is added to the system from an external source.

**PROCESS 4-1 :** The cycle completes with this process, which is a constant volume heat transfer from the working fluid to the regenerative matrix. During this process both pistons move simultaneously to transfer the working fluid (at constant volume) back, through the regenerative matrix from the expansion space, to the compression space. In passage through the matrix, heat is transferred from the working fluid to the matrix, so that the working fluid decreases in temperature, and emerges at  $T_C$  into the compression space. Heat transferred in the process is contained in the matrix, for

transfer to the gas in process 2-3 of the subsequent cycle.

If the heat transferred in process 2-3 has the same magnitude as in process 4-1, then the only heat transfers between the engine and its surroundings are (a) heat supply at  $T_{max}$  and (b) heat rejection at  $T_C$ . This heat supply and heat rejection at constant temperature satisfies the requirement of the second law of thermodynamics for maximum thermal efficiency, so that the efficiency of the stirling cycle is the same as the Carnot cycle , i.e.  $\eta = (T_E - T_C) / T_E$ . The principal advantage of the stirling cycle over the Carnot cycle lies in the replacement of two isentropic processes by two constant volume processes, which greatly increases the area of the P-V diagram. Therefore, to obtain a reasonable amount of work from the stirling cycle, it is not necessary to resort to the impractically high pressures and swept volumes, as in the Carnot cycle.

## 2.2 Comparison with Carnot Cycle

A comparison of the P-V and T-S diagrams of a Carnot and Stirling cycle, between given limits of pressure, volume and temperature, is shown on (Figure 2.2). The shaded areas 5-2-3 and 1-6-4 on the P-V diagram represent the additional work available in Stirling cycle by substituting constant volume processes for isentropic processes of Carnot Cycle. On the T-S diagram, the isothermal processes (1-5 and 3-6) of the Carnot cycle are extended to process 1-2 and 3-4 in the Stirling cycle respectively, so that the quantities of heat supplied and heat rejected in the Stirling cycle are increased in the same proportion as the available work. The fraction of supplied heat which is converted to work i.e. thermal efficiency, is same in both cycles.

## 2.3 Practical Regenerative Stirling Cycle

### 2.3.1 Theoretical Assumptions

Following assumptions are made in the theoretical description of the ideal Stirling cycle :

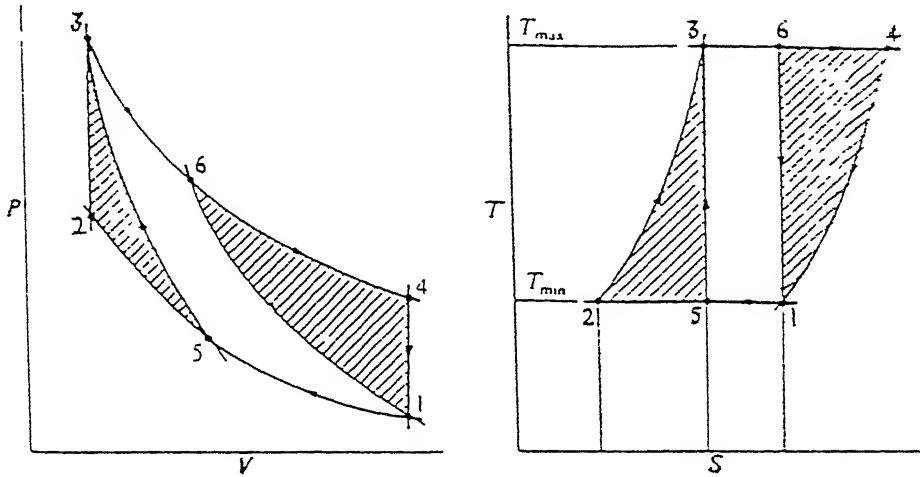


Figure 2.2: Stirling versus Carnot cycle (Walker, G. *et al* [4]).

1. All processes were assumed to be thermodynamically reversible and also the processes of compression and expansion were isothermal. It implies infinite rates of heat transfer between the cylinder walls and working fluid.
2. During the processes of expansion and compression all the working fluid was either in the compression or expansion space, so that the effects of any voids in the regenerative matrix, clearance space, or pockets in the cylinder were neglected.
3. The two pistons are assumed to move in some discontinuous fashion to achieve the prescribed working fluid distribution.
4. All aerodynamic and mechanical friction and leakage effects are neglected.
5. Regeneration is supposed to be perfect, which implies an infinite rate of heat transfer between the working fluid and regenerative matrix, and an infinite heat capacity of the regenerative matrix.

### 2.3.2 Modifications in Practical Stirling Engine

Most practical Stirling engines have the following heat exchangers which differ them from the simplified representations of Stirling engines.

**Regenerator :** The regenerator is the characteristic of the Stirling engine, and with its help an ideal engine would have Carnot efficiency. A regenerator is a heat exchanger placed between hot and cold end of the engine. It is made by stacking together a number of fine wire to form a kind of metallic sponge through which the gas must flow during the displacer action. Heat is transferred from the working fluid to regenerative matrix when hot gases passes through it. The same heat is returned to the working the passages of the cold fluid through the regenerator towards the heater and expansion space.

**Heater and Cooler :** The second difference between practical stirling engines and the simplified one is that most modern engines operates at high speed typically 3000 rpm and the gas in the cylinder has no time to come in equilibrium with the cylinder walls, cylinder head, and piston crown. Infact, very little time is available for heat transfer and the conductivity of even hydrogen and helium is so low, that the gas behavior in the cylinder is often very nearly adiabatic. It is, therefore, usual to fit an external heater and cooler, designed with a high surface area and narrow passages that minimize the heat conduction path though the gas, to add the heat absorbed during the expansion phase and to reject the heat generated during the compression. The net difference between the heat added and heat rejected is the mechanical work available from the power piston.

**Sinusoidal Motion :** In almost all real engine the piston movements are not sequential and discontinuous. In most cases the piston move simultaneously and almost sinusoidally. With sinusoidally moving pistons the pressure variations and mass of gas in each of the spaces will also vary almost sinusoidally. With a phase difference of  $90^\circ$ , When displacer is at top dead center, leaving most of the gas at the cold end of the engine, then power piston is the middle of its compression

stroke. Similarly, displacer is at bottom dead center, having transferred some of the gas into the hot space, then power piston is in the middle of the expansion stroke.

## 2.4 Mechanical Arrangements

Stirling engine have two spaces called cold spaces and hot spaces. These spaces are coupled through a regenerative heat exchanger and two auxiliary heat exchangers. One is the water cooled cooler on the cold space side of the regenerator and other one is the heater on the hot space side of the same. All these elements can be arranged in different mechanical arrangements. Stirling engines are either *single acting* or *double acting*. Single acting machines can further be classified in two types of mechanical arrangements namely : two piston machines i.e.  $\alpha$ -configuration (refer Figure 2.3(c)) or displacer-piston machines. Further, displacer-piston arrangement can be arranged in two ways : (a) piston and displacer are in the same cylinder i.e.  $\beta$ -configuration, (refer Figure 2.3(a)) and (b) piston and displacer operates in different cylinders i.e.  $\gamma$ -configuration (refer Figure 2.3(b)).

Pistons can be differentiated from the displacers with regard to pressure and temperature. Pistons are high pressure difference and low temperature difference devices used for compression and expansion while displacers are low pressure difference and high temperature difference devices used for displacing working fluid between hot and cold spaces.

## 2.5 Control Systems

The power output of a Stirling engine depends on a variety of parameters like temperature ratio, dead volume ratio, mean cyclic pressure, the speed of engine, the load speed characteristic etc..

**Temperature Control :** An increase in the coolant flow can decrease the compression space temperature and so effect an increase in power output and improvement

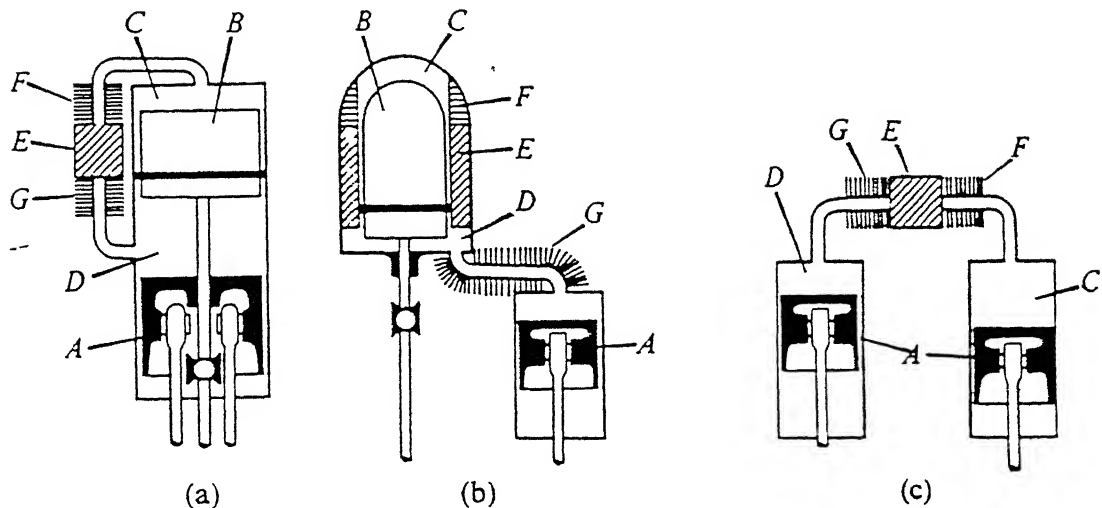


Figure 2.3: Different Stirling engine configurations : (a)  $\beta$  - configuration, (b)  $\gamma$  - configuration and (c)  $\alpha$  - configuration (West, C.D. [5]).

is linear but slight. The response of the Stirling engine to increased fuel supply is slow due to high thermal inertia of the hot parts. It takes from fifteen to thirty seconds or more for the increase in fuel supply to be reflected in the performance of the engine.

**Pressure Control :** Pressure of the working fluid can be controlled by having a reservoir of high pressure gas that can be admitted to working space through a control valve. Approximately, power output is a linear function of pressure but combined various practical effects make power/pressure characteristic non-linear where power increase occurs at diminishing rate. Thermal efficiency increases with pressure to a maximum then decreases (refer Figure 2.4).

**Speed Control :** Theoretically, power output of a Stirling engine is directly proportional its speed but practically, power increases at a progressively diminishing rate

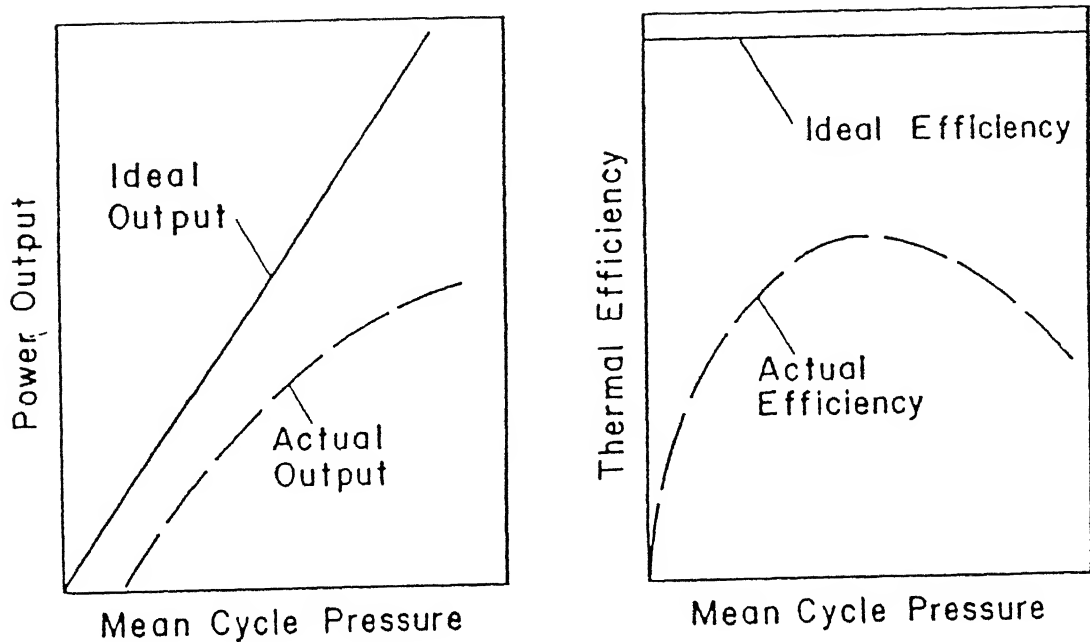


Figure 2.4: Effect of pressure increase on the power output and efficiency of a Stirling engine (Walker, G. *et al* [4]).

due to fluid friction losses, thermal saturation of regenerative matrix and decreased effectiveness of the heat exchangers (refer Figure 2.5).

**Stroke Control :** Power output of a Stirling can be effectively controlled through variation in the stroke of one or both the reciprocating pistons or displacers. Change in the pressure ratio is proportional to change in stroke and consequently volume ratio ( refer Figure 2.6).

**Dead Volume Control :** An increase in the dead volume reduces amplitude of cyclic pressure while mean pressure more or less remains constant. It reduces pressure ratio and consequently power output ( refer Figure 2.7).

**Phase Angle Control :** In a Stirling engine volume variation in the expansion space lead the volume variations in the compression space by phase difference  $\alpha$  to ensure that heat is absorbed by the expansion space and rejected from compression space.



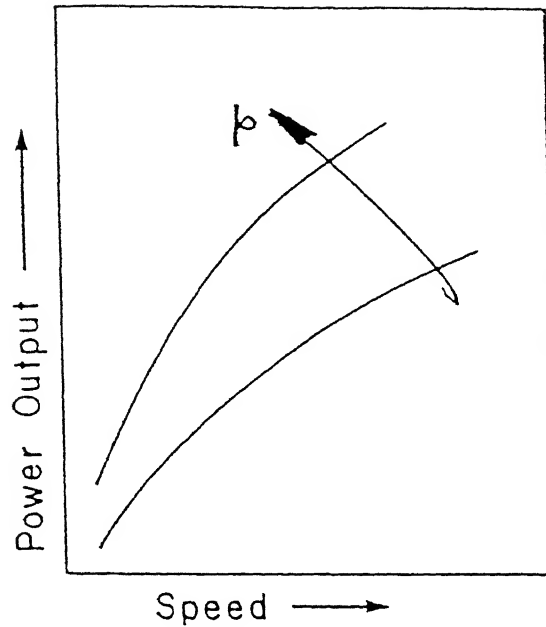


Figure 2.5: Power output of Stirling engine as a function of the speed (Walker, G. *et al* [4])

Power output increases with an increase in phase angle approaching maximum value at  $90^\circ$  and decreasing thereafter. Increase in phase angle beyond  $180^\circ$  causes reversal in the roles of two spaces (refer Figure 2.8).

## 2.6 Limitations

Each of the unique characteristics of the Stirling cycle also raises difficulties that are not shared by other common heat engines. They are as follows

- The engine thermal efficiency is dependent on the maximum cycle temperature so to sustain a reasonable thermal efficiency it is necessary to use relatively **expensive materials for the hot parts** like stainless steel or high temperature alloys. The continuous high temperature of the heat source promotes materials problems that are absent from an internal combustion engine, where the intermittent combustion

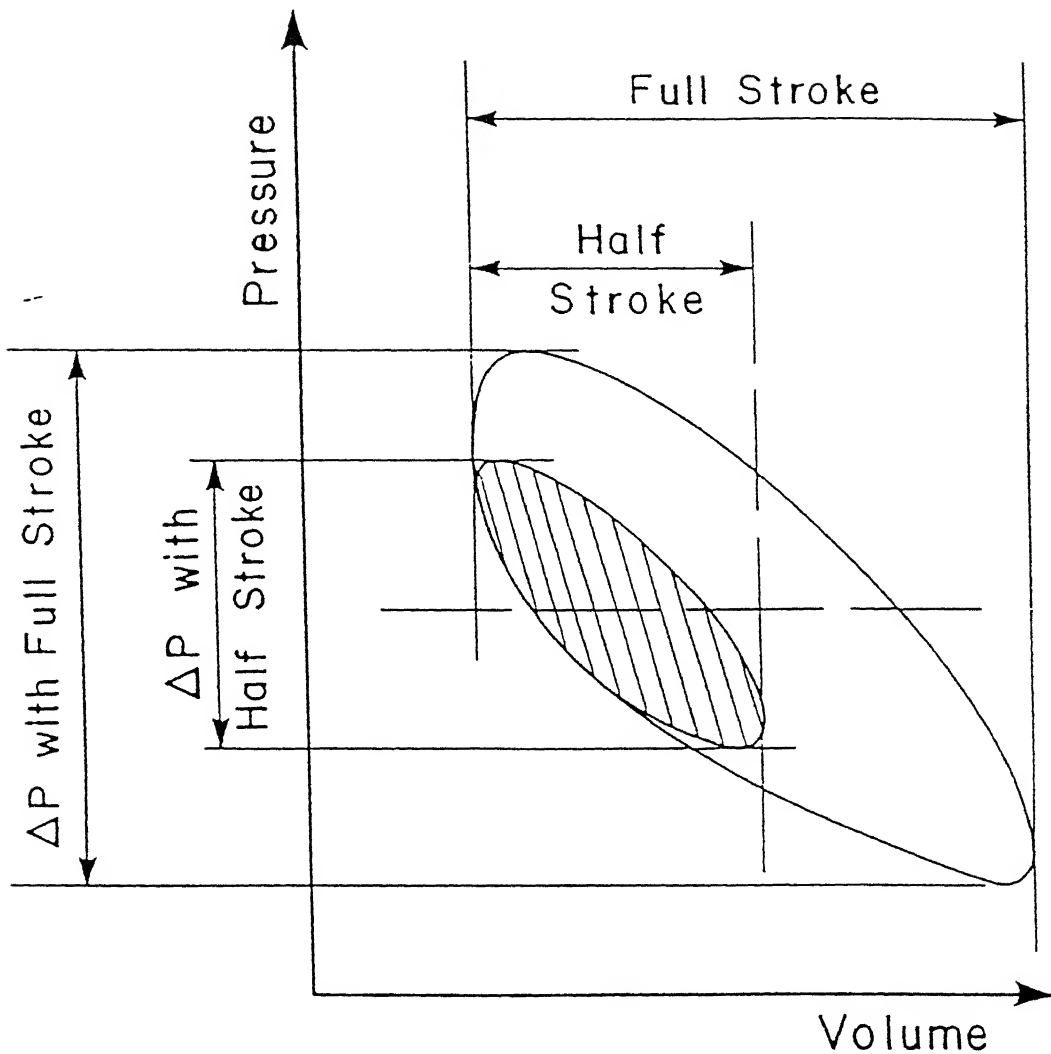


Figure 2 6. Effect of Stroke length variation on pressure ratio and power output in a Stirling engine (Walker, G. *et al* [4])

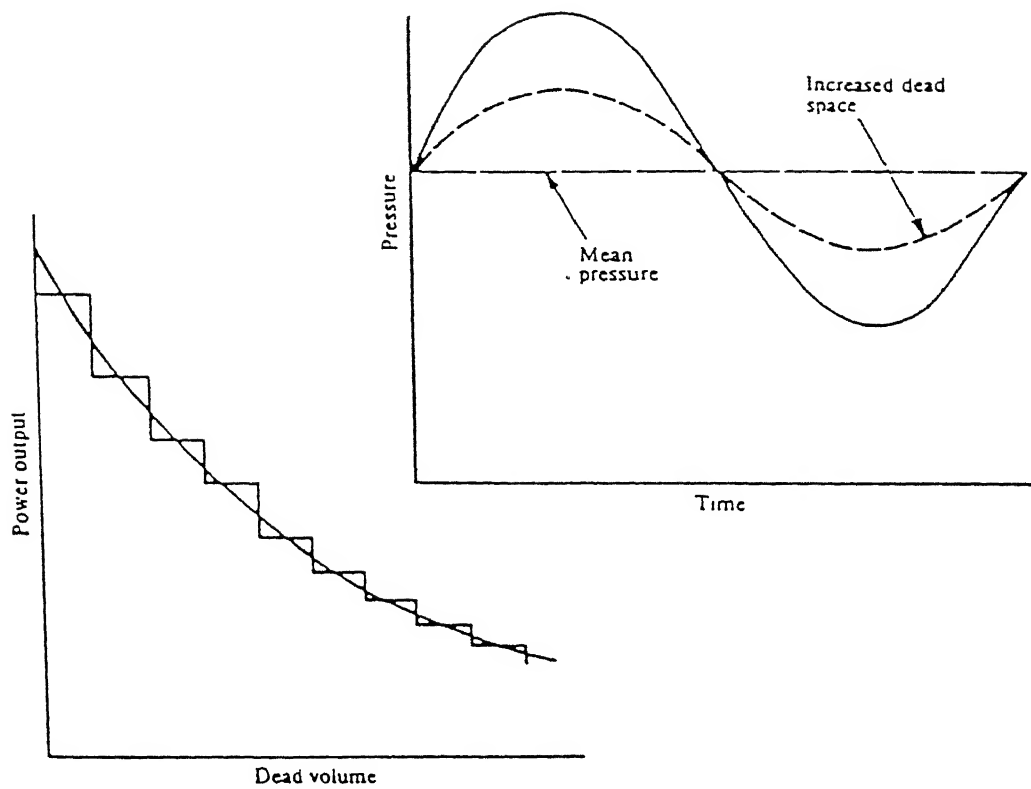


Figure 2.7: Effect of dead volume on power output in a Stirling engine (Walker, G. *et al* [4]).

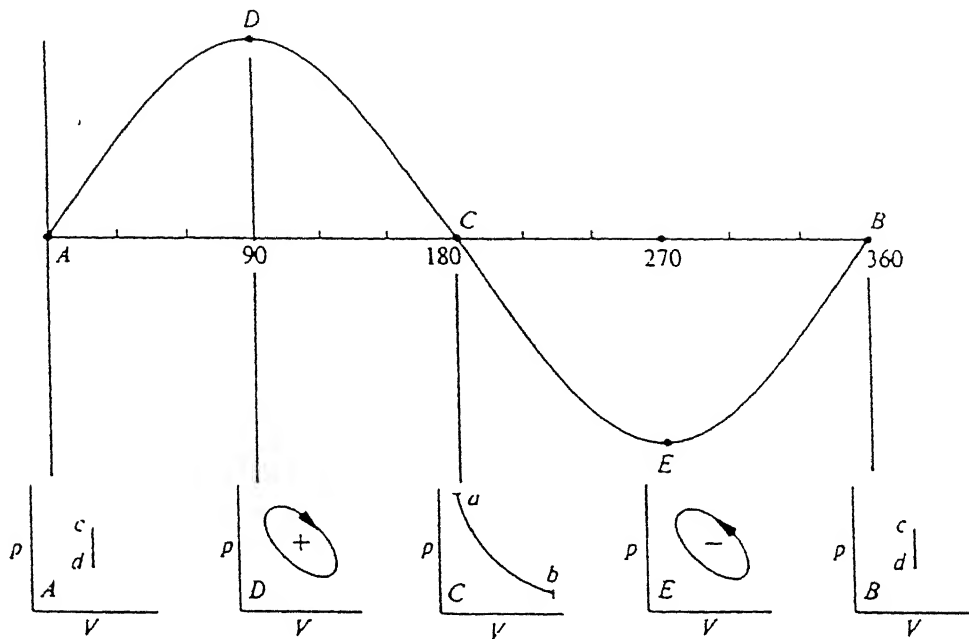


Figure 2.8: Effect on power output of a Stirling engine due to variation in phase angle  $\alpha$  (Walker, G. *et al* [4]).

process gives little time for heat transfer between the hot gas.

- The cooling load in the Stirling engine is heavier than an I.C. engine as there is no emission of exhaust gases and the working fluid is permanently contained in the cylinder. It requires a **heavy load cooler** having a thermal capacity approximately twice that of the I.C. engines of comparable power output. It increases the cost of Stirling engine.
- Stirling engine costs twice that of a diesel engine for same power output
- To retain the working fluid under high pressure from one cycle to another demands **an effective sealing system** around the working space. In air engines sealing is not a problem because it is a heavy gas. Use of light gas in high speed , highly pressured Stirling engine poses difficult sealing problems.

- The customary form of engine control, regulation of the fuel supply, is routinely used for most Stirling engines but the engine response is very low because of thermal inertia. So a supplementary control system(Dead space, Mean pressure or Phase angle) is necessary to provide rapid response and these control system are complex.

Next chapter presents analysis of Stirling engine.

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# Chapter 3

## Stirling Engine Analysis

The classical analysis of the operation of Stirling engines is due to the Schmidt(1861). The theory provides for harmonic motion of the reciprocating elements but retains the major assumptions.

### 3.1 Schmidt Analysis

Following are the principal assumptions of Schmidt analysis :

- 1 Regenerative process is perfect,
2. Instantaneous pressure is the same throughout the system,
- 3 Working fluid obeys the characteristic gas equation,
4. There is no leakage and the mass of working fluid remains constant,
5. Volume variations in the working space occur sinusoidally,
6. There are no temperature gradients in the heat exchangers,
- 7 Cylinder wall and piston temperature is constant.
8. There is a perfect mixing of the cylinder contents,
- 9 Temperature of the working fluid in the ancillary spaces is constant,

10. Speed of the engine is constant, and
11. Steady state conditions are established.

The volumes of expansion space (refer Figure 2.1(b)) ( $V_e$ ), compression space ( $V_c$ ) and dead space ( $V_d$ ) are given by the following equations respectively :

$$V_e = \frac{1}{2}V_E[1 + \cos\phi] \quad (3.1)$$

$$V_c = \frac{1}{2}V_C[1 + \cos(\phi - \alpha)] \quad (3.2)$$

$$V_c = \frac{1}{2}kV_E[1 + \cos(\phi - \alpha)] \quad (3.3)$$

$$V_D = XV_E \quad (3.4)$$

While mass of working fluid in expansion space ( $M_e$ ), compression space ( $M_c$ ) and dead space ( $M_d$ ) are given by .

$$M_e = p_e V_e / RT_e; M_c = p_c V_c / RT_c; M_d = p_d V_d / RT_d$$

respectively. Since total mass of the working fluid ( $M_T$ ) remains constant, it is given by :

$$M_T = p_e V_e / RT_e + p_c V_c / RT_c + p_d V_d / RT_d = KV_E / 2RT_c \quad (3.5)$$

If the instantaneous pressure  $p$  remains same throughout and instantaneous cold space and hot space temperature  $T_c$  and  $T_e$  respectively are equal to  $T_C$  and  $T_E$  then substituting for volumes in the previous equation.

$$\frac{K}{p} = T_C / T_E (1 + \cos\phi) + k[1 + \cos(\phi - \alpha)] + 2V_D T_C / (V_E T_D) \quad (3.6)$$

If the temperature variation in the dead space is linear in the axial direction, then the dead space mean temperature ( $T_D$ ),

$$T_D = T_C + \frac{1}{2}(T_E - T_C) = (1 + T_E/T_C)T_C/2$$

Since  $T_C/T_E = \xi$  then, from Equation 3.6,

$$\frac{K}{p} = \xi(1 + \cos\phi) + k[1 + \cos(\phi - \alpha)] + 2S \quad (3.7)$$

Where reduced dead volume  $S = 2X\xi/\xi + 1$ . To simplify Equation 3.7 consider that

$$y = x\cos\phi + z\sin\phi \quad (3.8)$$

then  $y = r\cos(\phi - \beta)$ , where  $\tan\beta = z/x$  and  $z = r\sin\beta$  and  $x = r\cos\beta$  Since

$$r\cos(\phi - \beta) = r(\cos\phi\cos\beta + \sin\phi\sin\beta) = x\cos\phi + z\sin\phi$$

Equation 3.8 is similar in form to Equation 3.7 therefore, by analogy,

$$\begin{aligned} \frac{K}{p} &= [(\xi + k\cos\alpha)^2 + (k\sin\alpha)^2]^{\frac{1}{2}}\cos(\phi - \theta) + \xi + k + 2S \\ \frac{K}{p} &= (\xi^2 + 2\xi k\cos\alpha + k^2)^{\frac{1}{2}}\cos(\phi - \theta) + \xi + k + 2S \end{aligned} \quad (3.9)$$

Where  $\tan\theta = (k\sin\alpha)/(\xi + k\cos\alpha)$

Let  $A = (\xi^2 + 2\xi k\cos\alpha + k^2)^{\frac{1}{2}}$ ,  $B = \xi + k + 2S$  and  $\delta = A/B$

Then  $\frac{K}{p} = A\cos(\phi - \theta) + B$  and  $p = K/[B(1 + \delta\cos(\phi - \theta))]$

The instantaneous pressure  $p$  is

a) a minimum, when  $\phi = 0, i.e.(\phi - \theta) = 0$ ,

b) a maximum, when  $\phi = (\phi + \pi), i.e.(\phi - \theta) = \pi$

So,  $p_{min} = K/B(1 + \delta)$  and  $p_{max} = K/B(1 - \delta)$

Thus,



$$p = p_{max}(1 - \delta)/1 + \delta \cos(\phi - \theta) = p_{min}(1 + \delta)/1 + \delta \cos(\phi - \theta) \quad (3.10)$$

The pressure ratio

$$p_r = p_{max}/p_{min} = \left( \frac{1 + \delta}{1 - \delta} \right) \quad (3.11)$$

### 3.1.1 Mean Cycle Pressure

The mean cyclic pressure ( $p_{mean}$ ) is given by

$$p_{mean} = \left( \frac{1}{2\pi} \right) \int_0^{2\pi} p d(\phi - \theta)$$

$$p_{mean} = \frac{1}{2\pi} \int_0^{2\pi} [p_{max}(1 - \delta)/1 + \delta \cos(\phi - \theta)] d(\phi - \theta) \quad (3.12)$$

which can be resolved to

$$p_{mean} = p_{max} \left[ \frac{(1 - \delta)}{(1 + \delta)} \right]^{\frac{1}{2}} \quad (3.13)$$

### 3.1.2 Heat Transferred and Work Done

Since the processes of expansion and compression are isothermal the, the heat transferred  $Q$  is equal to the work done  $P$ , so

$$Q = P = \int p dv \text{ if } V = \frac{1}{2} V_E (1 + \cos \phi)$$

$$dV = -\frac{1}{2} V_E \sin \phi d\phi \quad (3.14)$$

and

$$p = p_{mean} [1 - \Delta \cos(\phi - \theta)] \quad (3.15)$$

where  $\Delta = 2\delta/[1 + (1 - \delta^2)^{\frac{1}{2}}]$

$$Q = -\frac{1}{2} p_{mean} \pi V_E \Delta \sin \theta \quad (3.16)$$

**Expansion space :** The variation in volume in the expansion space follows the equation :

$$V_e = \frac{1}{2}V_E(1 + \cos\phi)$$

so, heat transferred in the expansion space is given by

$$Q = \pi f p_{mean} V_E \delta \sin\theta / [1 + (1 - \delta^2)^{\frac{1}{2}}] \quad (3.17)$$

**Compression space :** The variation in volume of the compression space follows the equation :

$$V_c = \frac{1}{2}kV_E[1 + \cos(\phi - \alpha)] \quad (3.18)$$

so heat transferred in compression space is given by

$$Q_c = \pi f p_{mean} V_E k \delta \sin(\theta - \alpha) / [1 + (1 - \delta^2)^{\frac{1}{2}}] \quad (3.19)$$

Dividing Equation 3.19 by Equation 3.18

$$Q_c/Q = [k \sin(\theta - \alpha)] / \sin\phi = [( \cos\alpha - \sin\alpha ) / \tan\theta]$$

But  $\tan\theta = k \sin\alpha / (\xi + k \cos\alpha)$  and so  $Q_c/Q = -\xi$

The heat transferred in the expansion space is of opposite sign to that of heat transferred in the compression space and is numerically different by the temperature ratio  $\xi$ . By analogy, the work done in the two spaces has the same relationship,  $P_C = -\xi P_E$ . and the net power is

$$P = P_E + P_C = (1 - \xi)Q \quad (3.20)$$

So thermal efficiency

$$\eta = \frac{Q - \xi Q}{Q} = 1 - \xi = \frac{T_E - T_C}{T_E} \quad (3.21)$$

This corresponds to the Carnot efficiency.

### 3.1.3 Mass Distribution in the Machine

From the characteristic gas equation,

$$M = pV/RT$$

$$\text{Where } p = p_{mean}(1 - \delta^2)^{\frac{1}{2}}/[1 + \delta \cos(\phi - \theta)]$$

(a) The instantaneous mass of working fluid in the expansion space ( $M_e$ ) is given by

$$M_e = \frac{1}{2}V_E p_{mean}(1 - \delta^2)^{\frac{1}{2}}(1 + \cos\phi)/[RT_e(1 + \delta \cos(\phi - \theta))] \quad (3.22)$$

The rate of change of working fluid in the expansion space is  $dM_e/d\phi$ .

(b) The instantaneous mass of working fluid in the compression space ( $M_c$ ) is given by

$$M_c = \frac{1}{2}kV_E p_{mean}(1 - \delta^2)^{\frac{1}{2}}(1 + \cos(\phi - \alpha))/[2RT_c(1 + \delta \cos(\phi - \theta))] \quad (3.23)$$

The rate of change of working fluid in the compression space is  $dM_c/d\phi$ .

(c) The instantaneous mass of working fluid in the dead space ( $M_d$ ) is given by

$$M_d = XV_E p_{mean}(1 - \delta^2)^{\frac{1}{2}}/[RT_d(1 + \delta \cos(\phi - \theta))] \quad (3.24)$$

and the rate of change of working fluid in the dead space is  $dM_d/d\phi$ .

Now  $dM_e + dM_c + dM_d = 0$ , so that total mass of working fluid  $M_T$  is constant. so.

$$M_T = V_E p_{mean}(1 - \delta^2)^{\frac{1}{2}}[\xi(1 + \cos\phi) + k1 + \cos(\phi - \alpha) + 2S]/[RT_c(1 + \delta \cos(\phi - \theta))]$$

and when  $\phi = 0$

$$M_T = V_E p_{mean}(1 - \delta^2)^{\frac{1}{2}}[\xi + k/21 + \cos\alpha + S]/[RT_c(1 + \delta \cos(\phi - \theta))] \quad (3.25)$$

### Heat Lifted and Engine Output

(a) Non-dimensional expressions based on the mass of the working fluid :

The heat lifted per unit mass of working fluid, combining Equations 3.17 and 3.25, is given by

$$Q_{mass} = Q/RT_c = \pi\delta\sin\theta(1+\cos\theta)/[(1-\delta^2)^{\frac{1}{2}}1 + (1-\delta^2)^{\frac{1}{2}}\xi + k/2(1+\cos\alpha) + S]. \quad (3.26)$$

Similarly the net engine output power per unit mass of working fluid is given by

$$P_{mass} = p/RT_c = (\xi - 1)Q_{mass}. \quad (3.27)$$

(b) Non-dimensional expressions in terms of characteristic pressures and volumes : may be devised as follows. The combined swept volume is given by

$$V_T = (V_E + V_C) = (1 + k)V_E$$

combining this with Equation 3.13 and 3.18, then

$$Q_{max} = Q/(p_{max}V_T) = [\pi(1-\delta)^{\frac{1}{2}}\delta\sin\theta]/[(1+k)(1+\delta)^{\frac{1}{2}}(1+(1-\delta^2))^{\frac{1}{2}}] \quad (3.28)$$

and,

$$P_{max} = (\xi - 1)Q_{max} \quad (3.29)$$

## 3.2 Optimisation of Design Parameters

It is obvious from the Schmidt cycle equations that the net cycle power and the thermal loads on the heat exchangers are direct linear function of engine speed (f), pressure of working fluid ( $p_{max}$ ), and size of the engine, expressed in terms of the total swept volume  $V_T$ . Optimisation of design on the basis of parameter  $P/p_{max}V_T$  results in a

configuration of maximum possible power within limits of the maximum pressure and combined swept volume. The maximum limits of the maximum pressure is an important criterion because it affects the weight and strength of engine while the combined swept volume  $V_T$  is indicative of size. The design of Stirling engine depends upon four design parameters  $x$ ,  $k$ ,  $\alpha$ ,  $\xi$ . These parameters must be determined at design stage. Only temperature ratio  $\xi$  can be varied after design. For  $X$ ,  $k$ ,  $\alpha$  variation, we will have to change the structure.

### 3.2.1 Consolidated Charts

Figure 3.1 was prepared using the power parameter  $P/p_{max}V_T$  as the basis for optimisation. Surfaces were generated for the value of  $P/p_{max}V_T$ , with different values of  $\alpha$  and  $k$  and constant values of  $\xi$  and  $X$ . The apex of the surface was established, and the apex values of  $P/p_{max}V_T$ ,  $\alpha_{opt}$  and  $k_{opt}$  were plotted on charts, are called consolidated charts.

The consolidated charts are recommended for use in the preliminary design of engine. First, decide the hot space temperature  $T_E$  by material consideration and appropriate dead volume ratio  $X$  (It should be minimum and reasonable). Temperature of cold space was  $T_C$  taken  $300^\circ \text{K}$  (For water or air cooling). At chosen hot space temperature, a vertical line is drawn through the three charts where it intersects the curve of constant dead volume ratio  $X$  so thus the value of  $k$ ,  $\alpha$ ,  $P_{max}$  may be determined from charts. Now with the knowledge of all four parameters  $\xi$ ,  $X$ ,  $k$ ,  $\alpha$ , it is possible to proceed to the detailed design of the engine by utilising design equation given earlier.

## 3.3 Practical Stirling Engine Analysis

The Schmidt analysis gives theoretically accurate picture of the power output possible but, it is too clumsy for practical research. Since a number of assumptions have been made in it, it would be unrealistic to expect that it gives accurate results practically. Complexity does not by itself guarantee confidence. In view of this, there was a need of a model which would give a more accurate estimate of the power available.

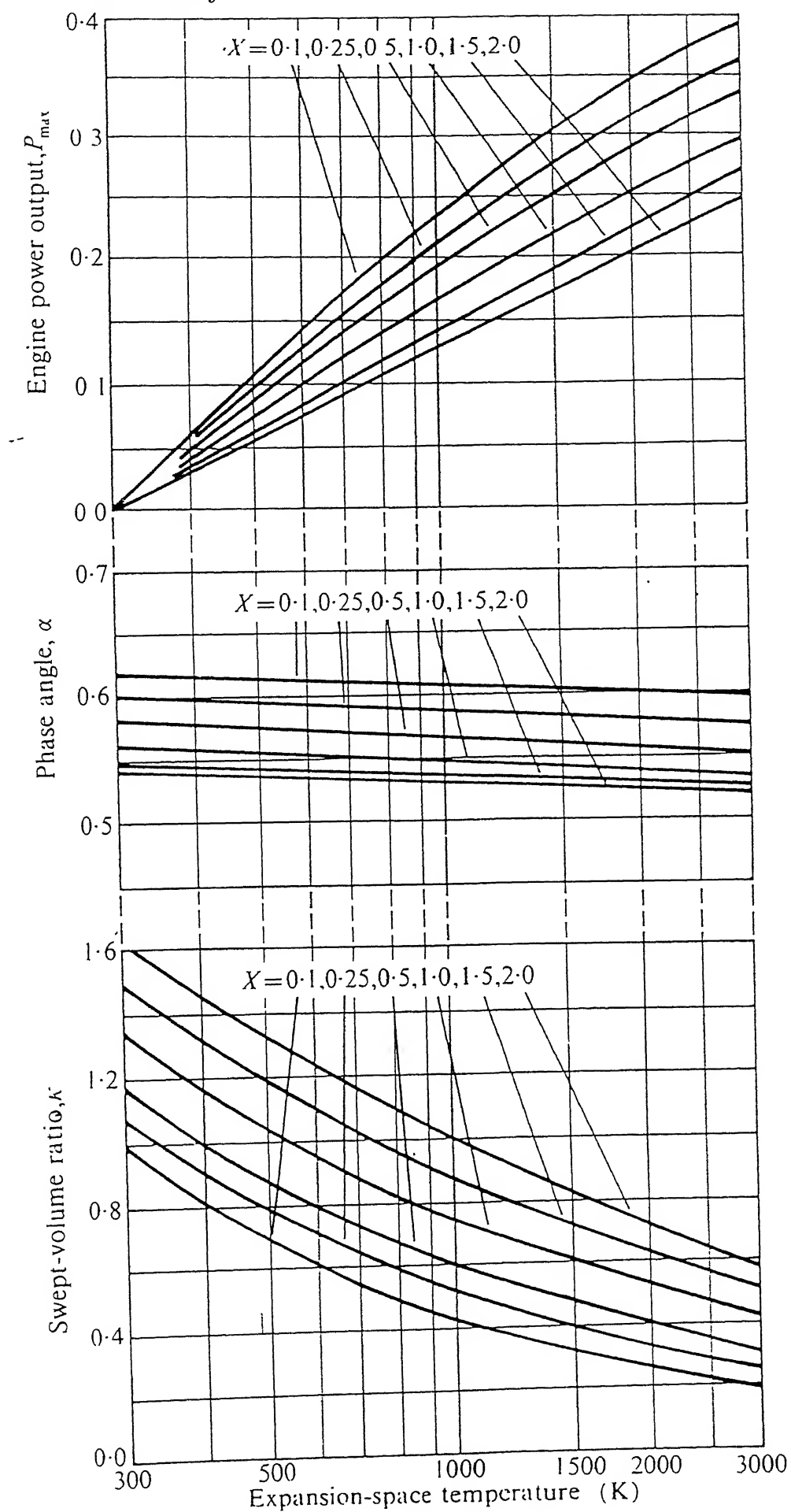


Figure 3.1: Consolidated charts for preliminary design (Walker, G. *et al.* [3])

### 3.3.1 C. D. West's Analysis of Stirling Engine

In ideal gas confined to a constant volume, (refer Figure {Fig : 1.2}) the pressure is simply proportional to the temperature. If the volume of the connecting ducts is negligible so that all of the gas is displaced between the hot and cold cylinder spaces, the resulting pressure change  $\delta p$  would be proportional to the temperature change  $(T_e - T_c)$ .

$$\delta p/p_{mean} = (T_e - T_c)/T_m \quad (3.30)$$

Actually, only a fraction  $V_e/V_m$  of the gas volume is displaced, the reminder being in the ducts and other spaces not reached by the piston and this reduces the pressure change accordingly

$$\delta p/p_{mean} = V_e/V_m (T_e - T_c)/T_m \quad (3.31)$$

One of our assumption will be that the mean temperature of the gas,  $T_m$ , is the average of the hot and cold space temperatures, i e.

$$T_m = 1/2(T_e + T_c) \quad (3.32)$$

The equation can be rewritten in terms of the hot and cold space temperature

$$\delta p/p_{mean} = V_e/V_m (T_e - T_c)/1/2(T_e + T_c) \quad (3.33)$$

In real machines, the pistons usually move in an approximately sinusoidal fashion. the exact nature of the motion depending on the kind of mechanical linkage that is used. For our present purposes it will suffice to assume that the piston does move sinusoidally and that the volume of the hot end of the displacer cylinder can be represented by

$$V_e(t) = \frac{1}{2}V_e(1 + \cos\omega t) \quad (3.34)$$

### Work Output

The differences between the force exerted on the power piston during the high pressure expansion phase and the smaller force exerted during the low pressure compression phase is the reason that more work can be done by the piston during expansion than must be put into the compression. In fact, if power piston volume is called  $V_o$ , then the work output during a single cycle,  $P$ , is simply  $V_o$ , times the difference between the high and low pressures. So, work is therefore:

$$P = V_o P_{mean} \frac{V_e}{V_m} \frac{(T_e - T_c)}{\frac{1}{2}(T_e + T_c)} \quad (3.35)$$

The work of the engine arises only from that component of the pressure oscillation due to the displacer movement that is not in phase with the power piston motion. In other way, if the power piston and the displacer are moving in a sinusoidal way, then only if there is a phase difference between the two components can there be any net work. If the volume in the power piston cylinder is varying with a phase that is  $\alpha$  radians behind the expansion space, i. e.,

$$V_o(t) = \frac{1}{2}V_o(1 + \cos(\omega t - \alpha)) \quad (3.36)$$

The component of displacer induced pressure that is not in phase with  $V_o$  is simply  $\delta p \sin\alpha$ . The work output is appropriately modified from equation(3.33):

$$P = V_o P_{mean} \frac{V_e}{V_m} \frac{(T_e - T_c)}{\frac{1}{2}(T_e + T_c)} \sin\alpha \quad (3.37)$$

We simply multiplied the maximum value of the pressure excursion,  $\delta p$ , and the total volume swept out by the power piston,  $V_o$ . Using the maximum value will somewhat over estimate the power, and a better result will be obtained by using some kind of mean



value for the pressure and volume excursion. The proper way to do this is to integrate the product of pressure and volume over the whole cycle:

$$P = \int P(t) dV_o(t) \quad (3.38)$$

Using the expressions for  $P(t)$  and  $V_o(t)$  obtained by equations (3.31), (3.32) and (3.33), the integral is a standard one and the result is

$$P = \pi/4 V_o P_{mean} \frac{V_e}{V_m} \frac{(T_e - T_c)}{\frac{1}{2}(T_e + T_c)} \sin \alpha \quad (3.39)$$

This is the same as equation (3.35) except that the factor  $\pi/4$ , which represents the effective average value of the variables instead of their maximum values. The equation gives work done per cycle to find out the work done per sec.

$$P = \pi/4 f V_o P_{mean} \frac{V_e}{V_m} \frac{(T_e - T_c)}{\frac{1}{2}(T_e + T_c)} \sin \alpha \quad (3.40)$$

This equation gives results that are within 5-10 percentage of the exact power developed by an engine.

Next chapter describes fabrication and analysis of  $\beta$  - Stirling engine.

## Chapter 4

# Fabrication and Analysis of $\beta$ - Stirling Engine Model

### 4.1 The Prototype Stirling Engine

Since the Stirling engine was a completely new subject to us, it was decided after discussion that the best course lay in producing a small prototype of an existing engine. This way a feel would be developed for the subject and visualization of problem would then be easier. Additionally a feel would be developed for manufacturing tolerances required for project

With view to this, the  $\beta$  - type of engine designed by Peter Meede of Germany was selected. Full but incomplete drawings of this engine were published in Model Engineer, October 1989 and were collected from British Council library in Calcutta. The technical descriptions of the fabricated model have been given in the following Section.

### 4.2 Details of the $\beta$ - Stirling Engine selected

#### 4.2.1 Technical Description

It is vertical inverted  $\beta$ - configuration single acting Stirling engine with surface type cooler with air cooled cooling space equipped with rectangular aluminium fins and circular type secondary fins.

**Crank mechanism :** It has Fixed and adjustable crank mechanism with 90° degree spacing between power and displacer piston crank. Crank is cantilevered of double ball bearings which supports turned steel flywheel.

**Combustion chamber :** It has Open combustion chamber fired by alcohol and transferring heat to turned brass hot end.

#### 4.2.2 Physical Dimensions

Table 4.1 summarizes the details of the operating parameters and physical dimensions of the fabricated model of  $\beta$  -Stirling engine.

Parameter	Value
Operating Speed	: 900 rpm
Pressure Ratio	: 1.41
Height of model	: 240 mm
Weight of model	: 2 Kg
Width of base	: 100 mm
Length of base	: 145 mm
Power piston bore	: 10 mm
Power piston stroke	: 16 mm
Displacer piston bore	: 16 mm
Displacer piston stroke	: 22 mm
Operating temperature	: 250° C

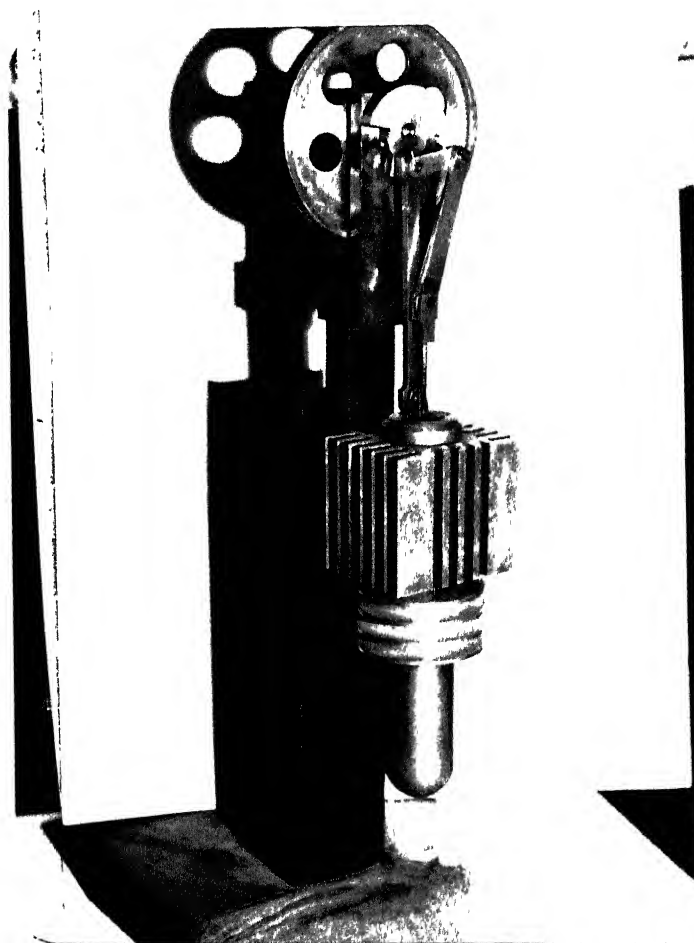
Table 4.1 Operating parameters and physical dimensions of the fabricated  $\beta$  - Stirling engine model.

#### 4.2.3 Material Requirements of the Fabricated Model

Table 4.2 presents the material requirements and physical dimensions of the different components of fabricated  $\beta$ -Stirling engine model, while corresponding drawings have been presented from Figure 4.2 to Figure 4.14

#### 4.2.4 Power Estimation of Model

Table 4.3 presents design parameters and performance of fabricated model.



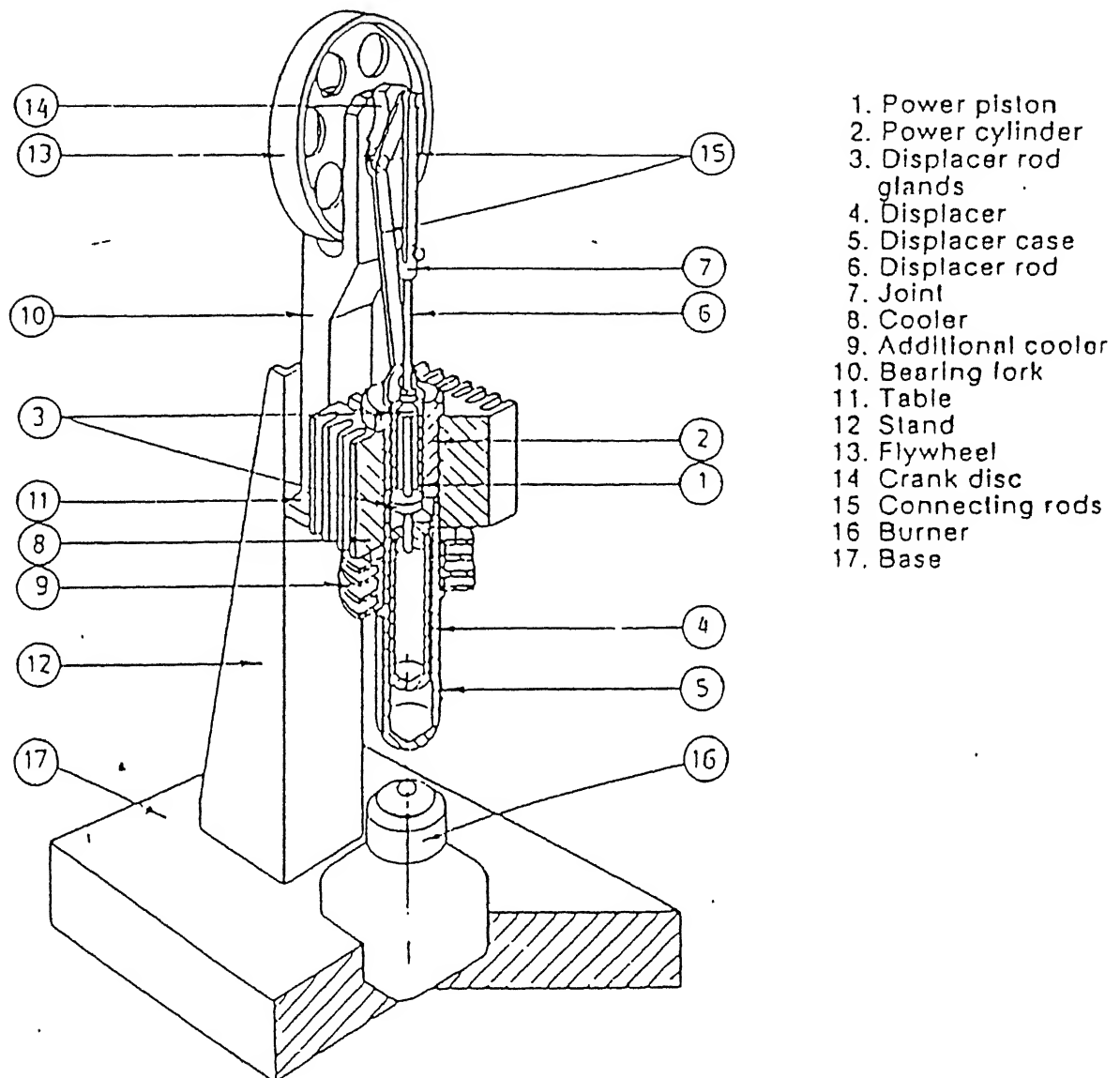


Figure 4.1: Fabricated  $\beta$ - Stirling engine model.

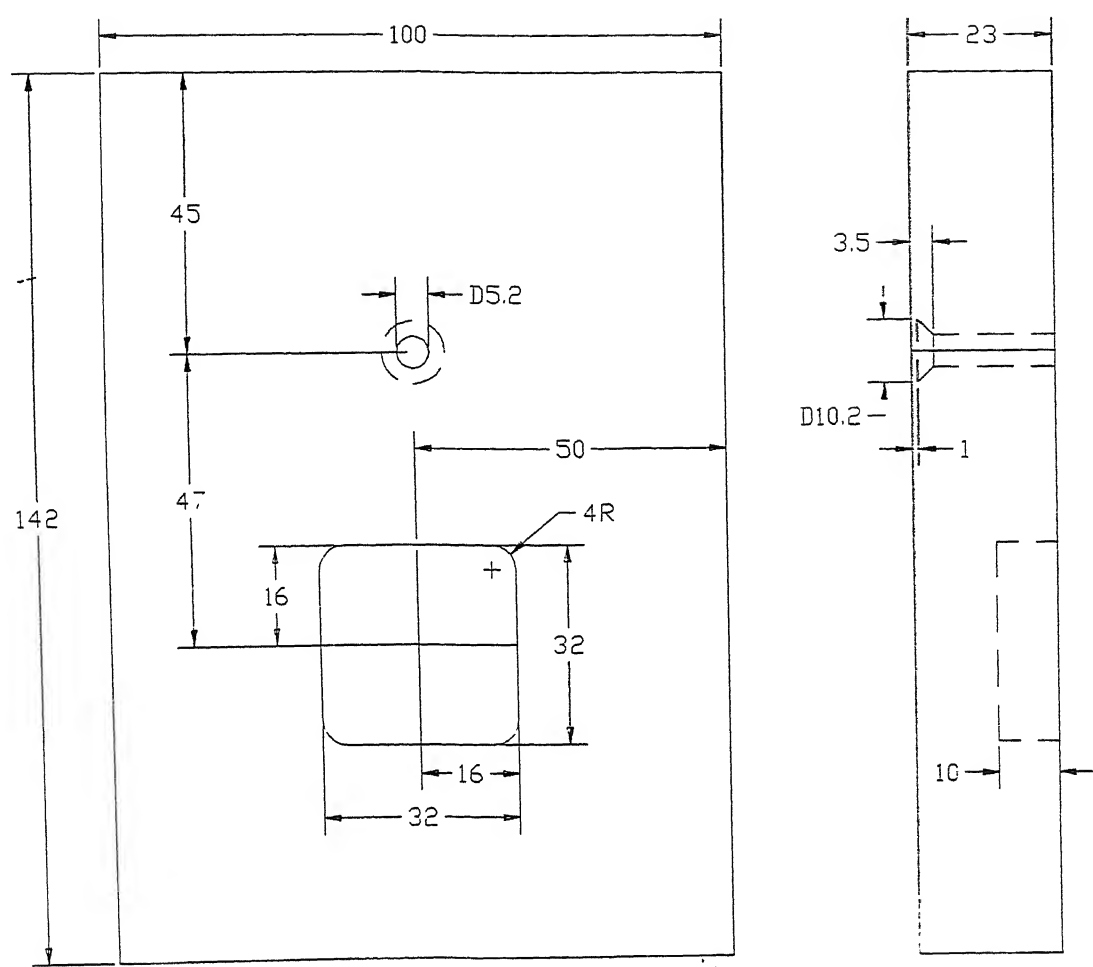


Figure 4.2. Engine base plate.

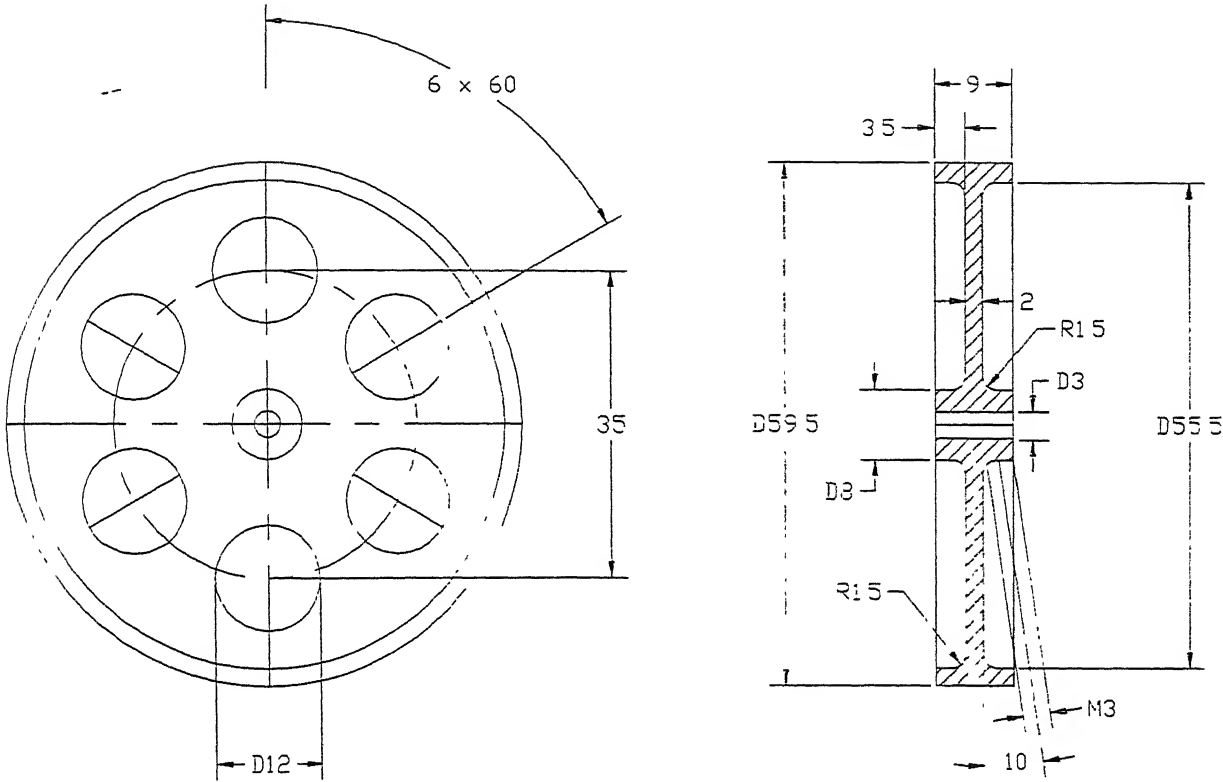


Figure 4.3: Flywheel.

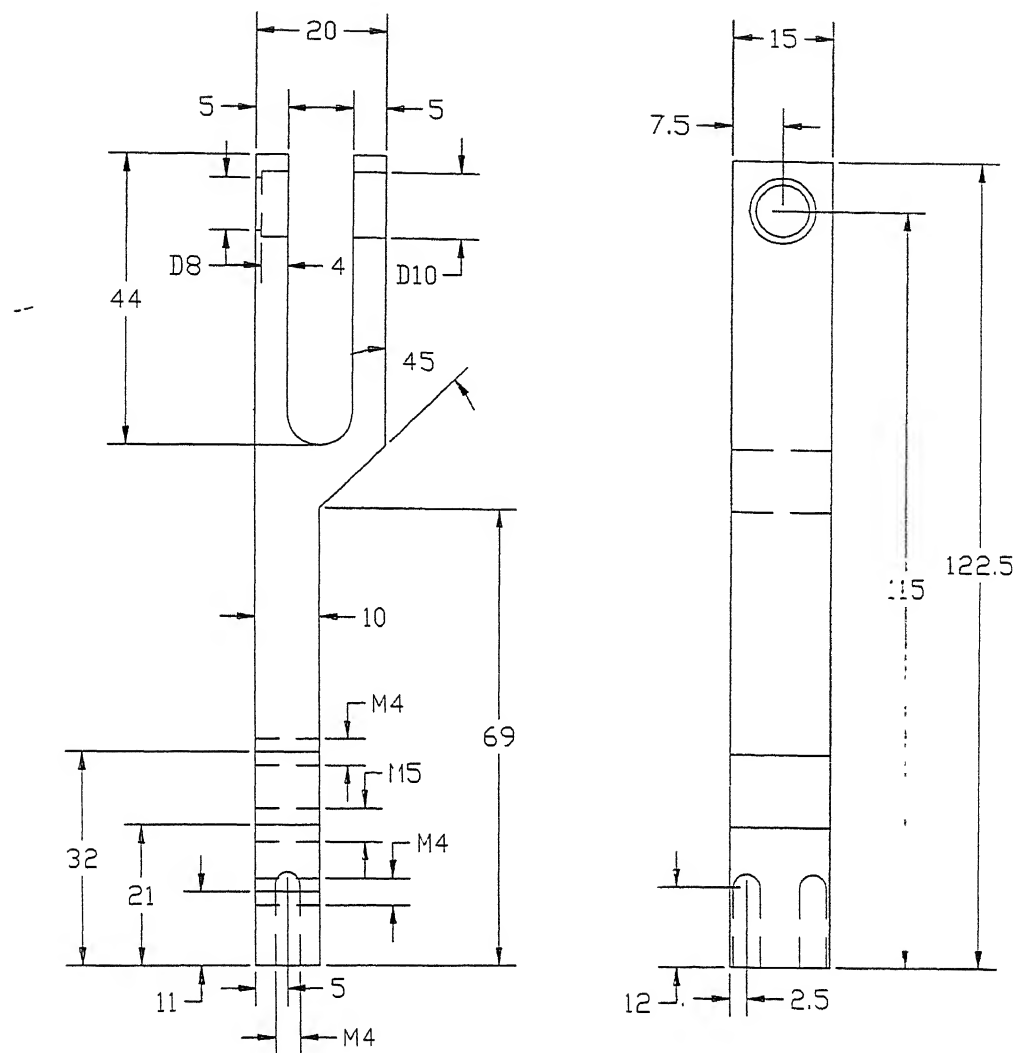


Figure 4.4: Bearing fork.



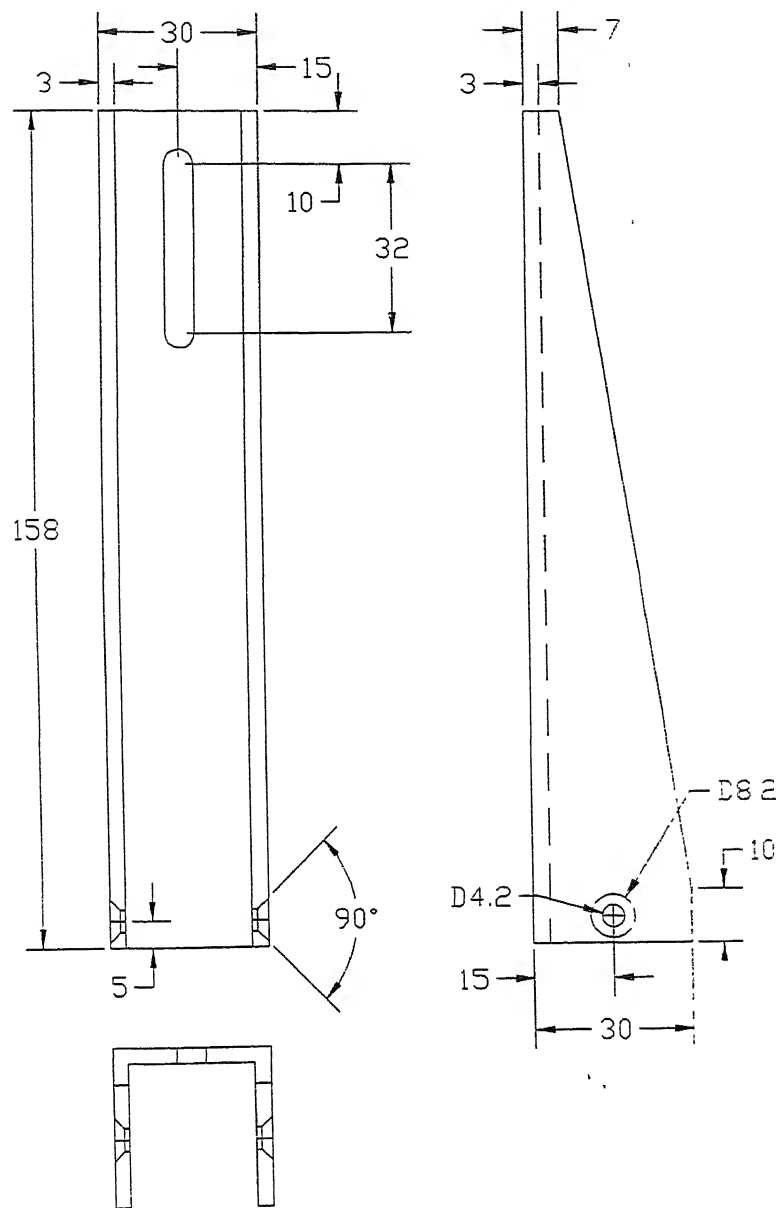


Figure 4.5: Engine stand.

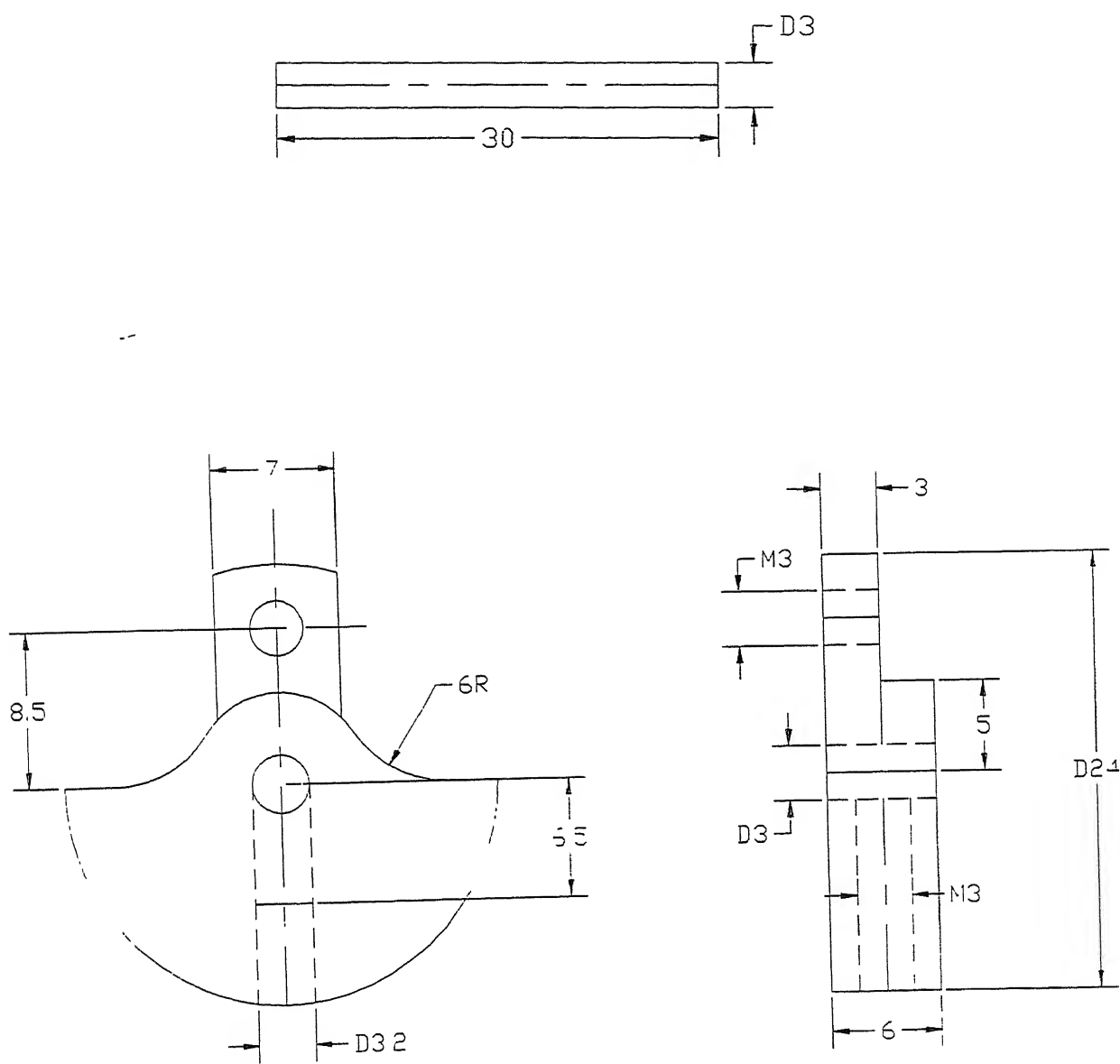


Figure 4.6: Crank disc and axle.

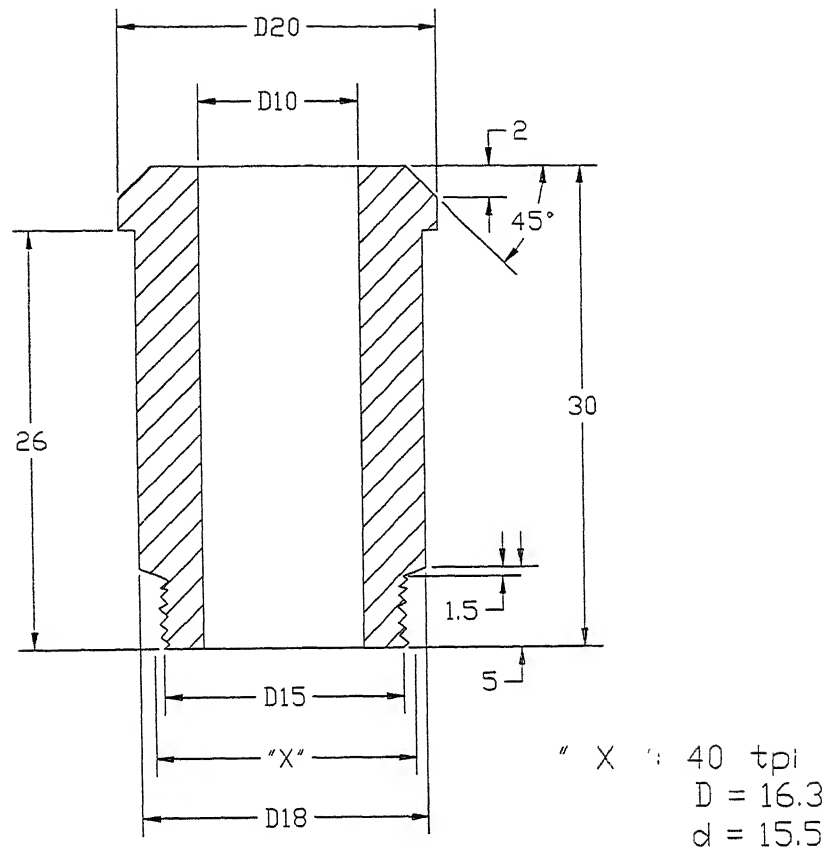


Figure 4.7. Power cylinder.

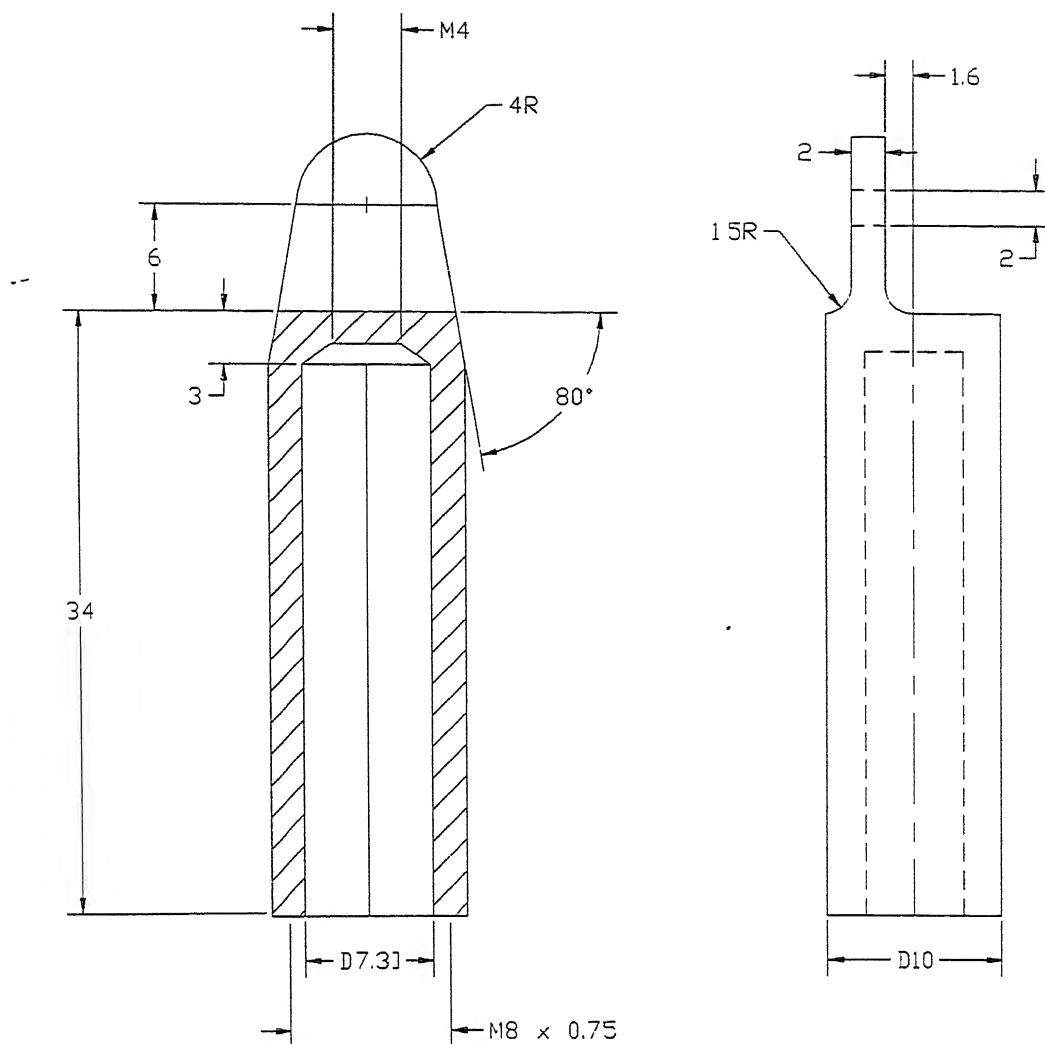


Figure 4.8. Power piston.

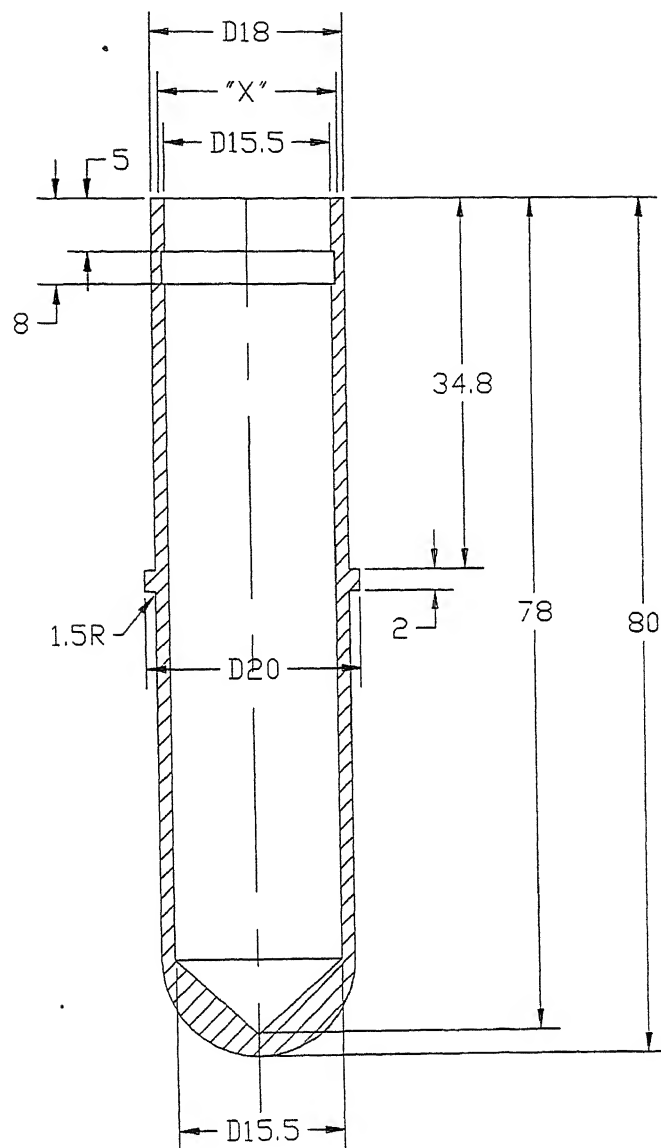


Figure 4.9 Displacer cylinder.

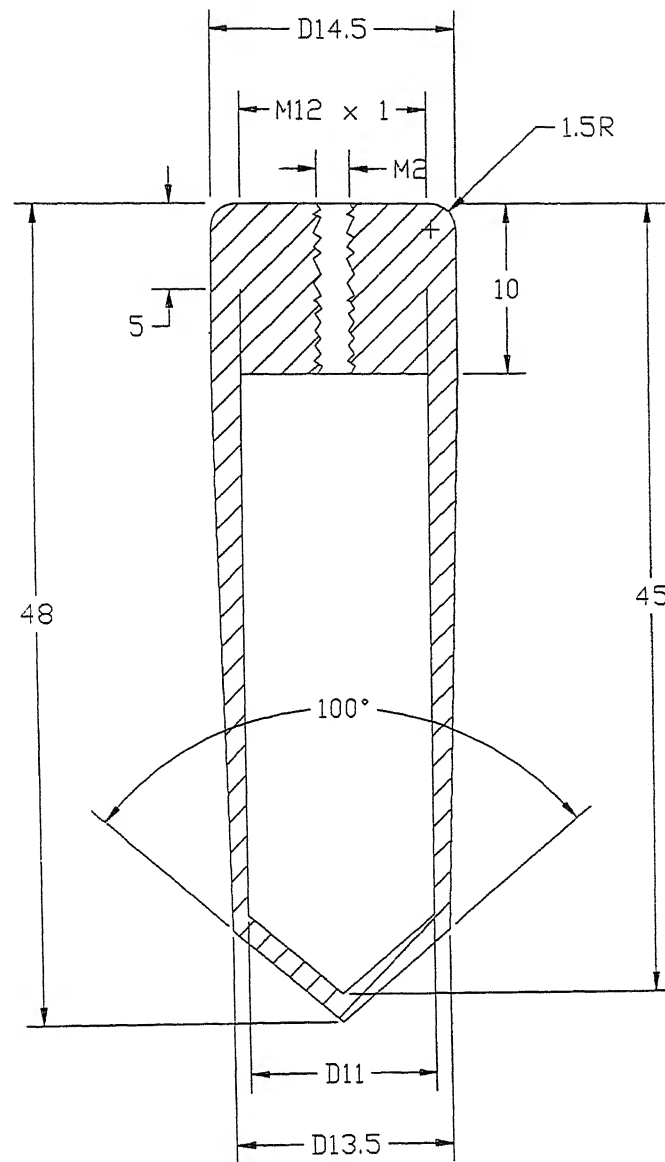


Figure 4.10: Displacer piston.

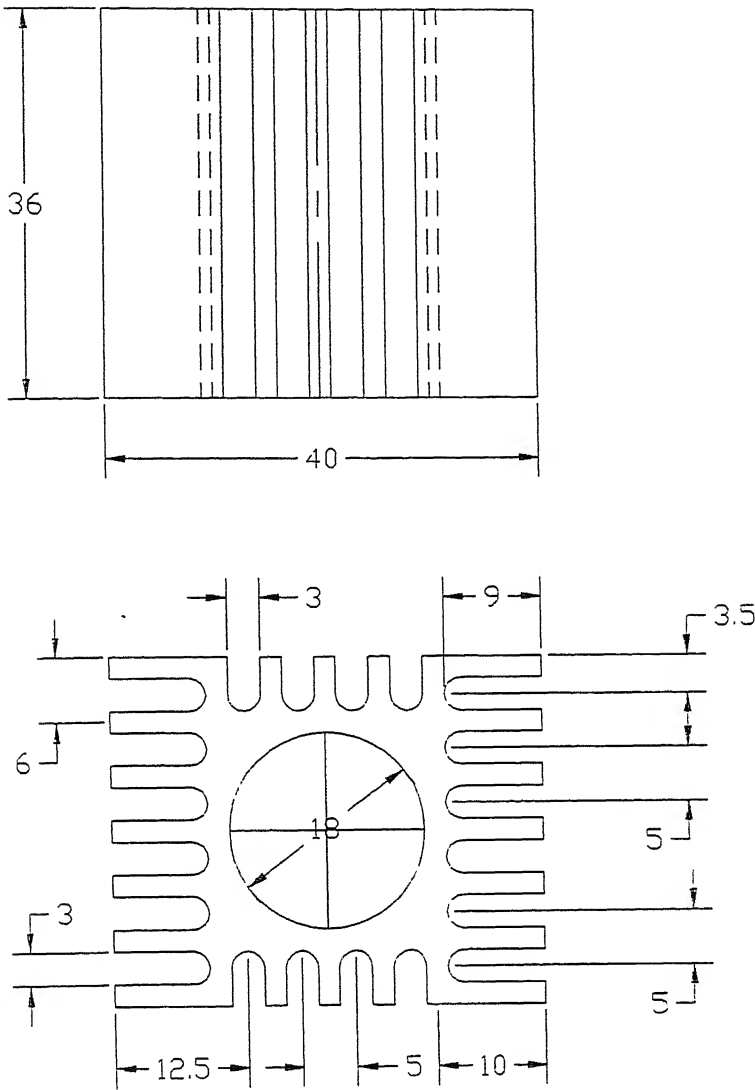


Figure 4.11: Rectangular cooler.

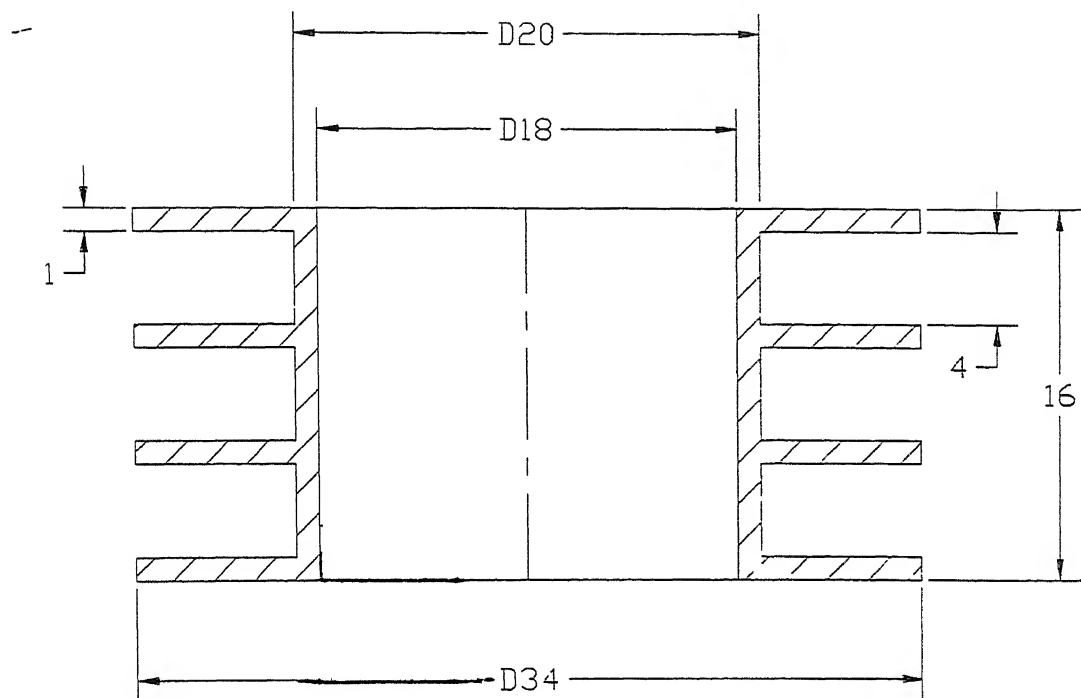


Figure 4.12: Circular cooler



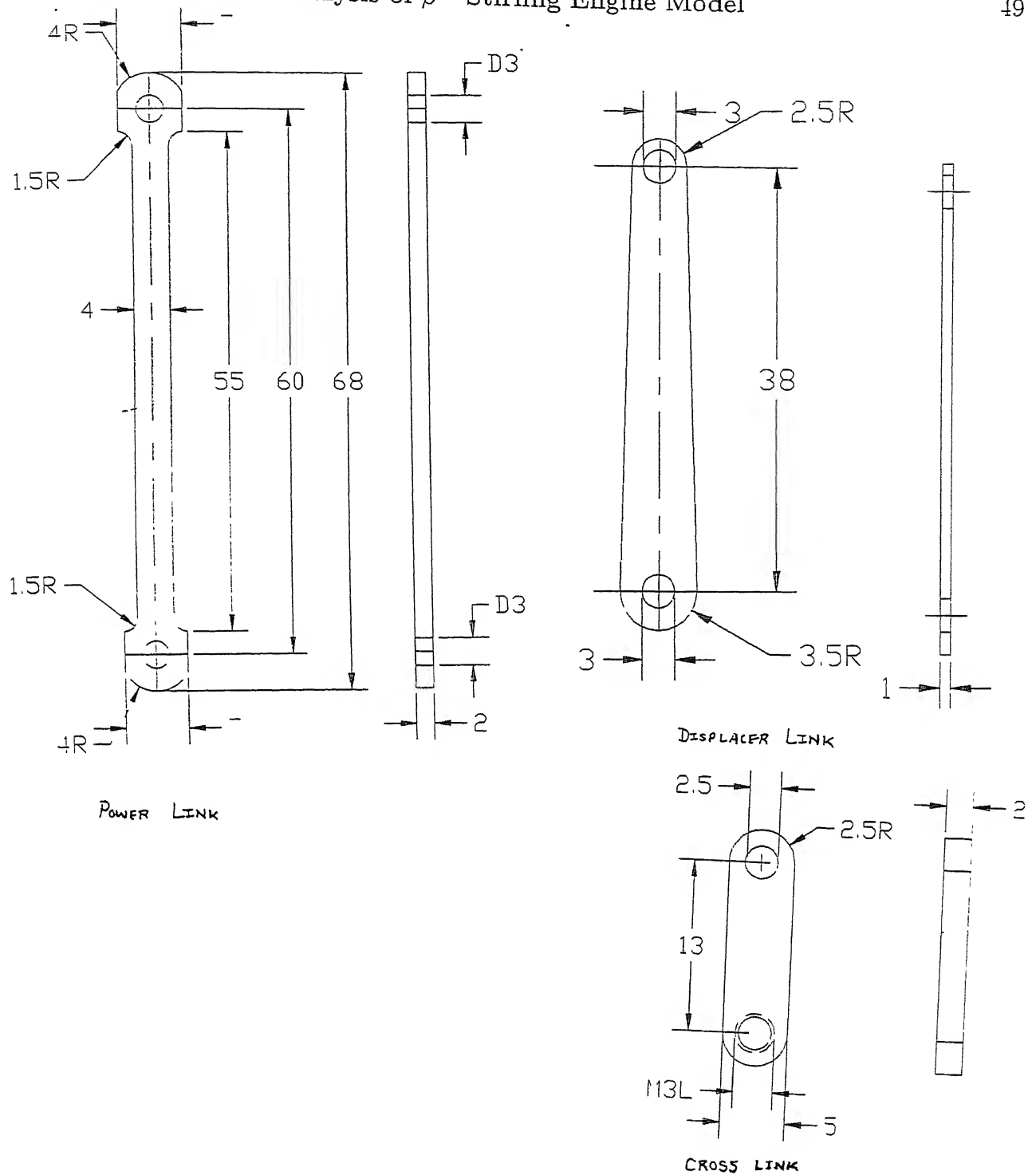


Figure 4.13: Connecting rod and connecting links.

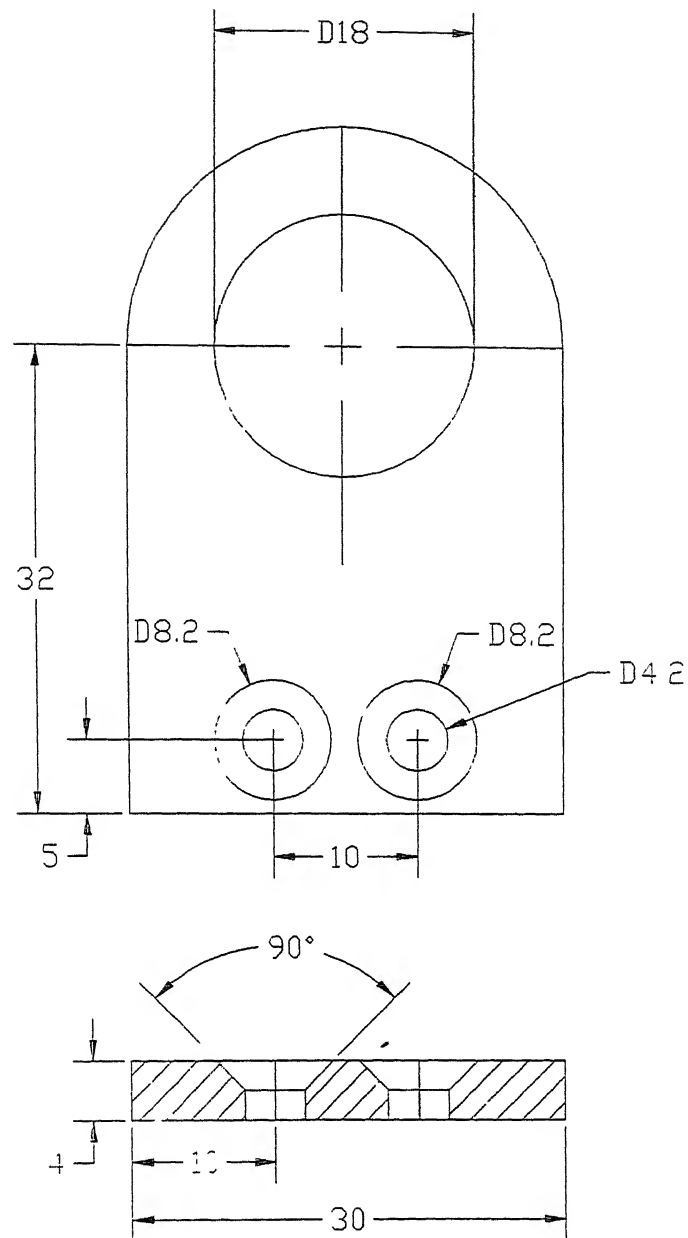


Figure 4.14: Table.

S.N.	Item	Material	Size	Reference Figure
1.	Engine base plate	Wooden Piece	145 x 100 mm	4.2
2	Flywheel	Mild Steel	$\phi$ 60 x 9 mm	4.3
3.	Bearing fork	Mild Steel	122.5 x 15 x 10 mm	4.4
4.	Stand	Mild teel	158 x 30 x 30 mm	4.5
5	Crank disc	Brass	$\phi$ 24 x 6 mm	4.6
6	Axle	Mild Steel	$\phi$ 3 x 30 mm	4.6
7.	Power cylinder	Brass	$\phi$ 20 x 30 mm	4.7
8.	Power piston	Mild Steel	$\phi$ 10 x 44 mm	4.8
9.	Displacer cylinder	Brass	$\phi$ 18 x 80 mm	4.9
10.	Displacer piston	Aluminium	$\phi$ 15 x 50 mm	4.10
11.	Rectangular cooler	Aluminium	40 x 36 x 36 mm	4.11
12.	Circular cooler	Aluminium	$\phi$ 34 x 16 mm	4.12
13.	Connecting rod	Mild Steel	$\phi$ 2 x 77 mm	4.13
14	Connecting link1	Brass	68 x 7 x 2 mm	4.13
15	Connecting link2	Brass	45 x 7 x 1 mm	4.13
16	Connecting link3	Brass	18 x 5 x 2 mm	4.13
17.	Table	Mild Steel	30 x 47 x 4 mm	4.14

Table 4 2 Raw material requirements of the fabricated model of  $\beta$  - Stirling engine.

Using Equation (3.40), the power developed by the fabricated model of  $\beta$  - Stirling engine at 900 rpm, 1 Bar minimum pressure and  $500^{\circ}K$  hot space temperature and  $300^{\circ}K$  cold space temperature, would be 0.258 Watts.

### 4.3 Features of Engine Design

#### 4.3.1 Drawbacks

Though it was described in the relevant publication that drawings were full and complete and that drawing of every part were available full technical detail which would enable one to make functional trouble free Stirling engine were infact not available. Follwing are the shortcomings of the existing drawings :

- Lack of surface finish specifications,
- Lack of tolerances,

Item	Parameter	Value	Reference
Expansion space	Stroke	22 mm	
	Bore	14 mm	
	Swept volume	3.38 cc	
	Temperature	500.0 K	
Compression space	Stroke	16 mm	
	Bore	10 mm	
	Swept volume	1.25 cc	
	Temperature	300.0 K	
Dead space	Volume	7.04	
Design parameters	$\xi$	.6	
	X	2.08	
	k	.369	
	$\alpha$	90°	
Pressures	Mean pressure	1.21	Eq : 3.13
	Maximum pressure	1.41 bar	Eq : 3.11
	Pressure ratio	1.41	
Speed		900rpm	
Engine performance	Heat input	.876 W	Eq : 3.17
	Power(Schmidt)	.35 W	Eq : 3.20
	Power(West)	.258 W	Eq : 3.40
	$\eta$	29.42	

Table 4.3. Design and performance parameters of fabricated  $\beta$  - Stirling engine.

- Lack of many small detail which are required to properly engineer the product,
- A design flaw was that the hot and cold surfaces are one above the other so the heat from the vertical fire will also affect cold surface which is just above the hot surface. This will obviously reduce the efficiency of the engine.

The above mentioned drawbacks are not meant to be criticism of published material but is being mentioned to a caution note to the future researchers so that they can take the appropriate steps. Our recommendation in this regard published below at the appropriate point.

Item	Parameter	Value	Reference
Expansion space	Stroke	22 mm	
	Bore	14 mm	
	Swept volume	3.38 cc	
	Temperature	500.0 K	
Compression space	Stroke	16 mm	
	Bore	10 mm	
	Swept volume	1.25 cc	
	Temperature	300.0 K	
Dead space	Volume	7.04	
Design parameters	$\xi$	.6	
	X	2.08	
	k	.369	
	$\alpha$	90°	
Pressures	Mean pressure	1.21	Eq . 3.13
	Maximum pressure	1.41 bar	Eq . 3.11
	Pressure ratio	1.41	
Speed		900rpm	
Engine performance	Heat input	.876 W	Eq . 3.17
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The above mentioned drawbacks are not meant to be criticism of published material but is being mentioned to a caution note to the future researchers so that they can take the appropriate steps. Our recommendation in this regard published below at the appropriate point.

### 4.3.2 Advantageous Points of Meede's Design

Strong point of Meede's engine design appear to be its simple design which enable to be made in a simple workshop provided craft skilled are high. The other attractive point that being a vertical engine, the piston friction are reduced to the minimum.

## 4.4 Fabrication of the Model

### 4.4.1 Manufacturing Difficulties

A number of problems were faced in the manufacture of Stirling engine which are as follows for the reference of future researchers.

**Bearing Design :** Bearing specified in the drawing (3 x 10 x 4); This size of bearing is not easily available. Infact, we were forced to use bearing of the size 4 x 13 x 5. As a result, the dimension of the components like fork end thickness, shaft diameter, flywheel bore and crank web inner bore had to be modified from 3 mm to 4mm.

**Fork :** During the assembly, it was found that fork prone to bending under the pressure of insertion causing misalignment and it is recommended that a packing piece be inserted into the gap of the fork to prevent this to happening. Infact, design was modified the design to a two piece fork. Further it was found that the drawing does not seem to provide a 90° alignment between crank pins. This cause malfunction of piston displacement of prototype. To overcome this type of problem in design it was decided to fix a pin between link and shaft.

**Left handed thread :** It should also be noted that the crank pin calls for a left hand thread. The left hand thread is important because it will insure the crank pin does not loose when the engine is running in the clockwise direction. Unfortunately many workshops are not equipped to manufacture the left hand thread and therefore the design was modified to ensure the problem is somehow overcome. Therefore. Araldite was used to join threaded surface of crank pin and corresponding bore.

After cleaning both surfaces with alcohol to avoid failure of araldite joint, as an ample precaution, a hole was drilled through to secure this item . After locating the link no. 1 another hole was drilled and another pin is fixed at appropriate angles It was found that the engine is extremely sensitive to friction caused by misalignment or any other lacunae in manufacturing.

#### 4.4.2 Assembly Problems

Initial trial of engine has shown that it was not functioning and the displacer and power piston were not functioning at correct phase angle. Thereafter steps were taken to reduce the lacunae in the drawings by the measures outlined above. After implementing these measures, the piston motion was achieved as desired above, but the engine was again stripped and reassembled to remove errors in the manufacturing. After several assemblies and disassemblies later, the engine failed to work. It was then decided that the engine will be stripped completely and measurements of surface dimensions, will be taken to know the exact reasons of non-functioning of the engine. The details of findings have been listed below:

- Displacer piston was touching to the displacer cylinder due to the inclination of connecting rod.
- There was much friction at seal joint due to the inclined connecting rod.

Another problem of this engine design is that cross head design is such that any deflection during manufacturing and assembly results in very high friction to such an extent that the engine will not run most probably because of the very high internal friction and low power output.

#### 4.4.3 First Running Test

After proper functioning between two pistons, the engine was run-in manually for about 60 minutes to make it free. It was then fired with burner but there was no functioning. Cold space became hot due to the heat transfer from hot space. So there was almost

temperature equalization between both space. this was the reason for no power out put. it seemed that frictional losses were more. It was also noticed that Teflon seal melted and came out due to the heat transfer from hot space to cold space.

#### 4.4.4 Frictional Force Measurement

- A thread was wrapped around the flywheel.
- Pan was attached for weight.
- Length of drop was measured.
- Time of drop was also noted.

S.No.	Weight	Time	Length of drop	Remarks
1.	50 gm	0.0 sec	0. m	Did not function
2.	100 gm	1.0 sec	0.84 m	O.K.
3.	80 gm	0.0 sec	0. m	Did not function
4.	90 gm	1.0 sec	0.84 m	O.K.

Table 4.4. Frictional force measurement experiment.

From these observations limiting frictional forces were known. Frictional torque with a flywheel of radius 3.0 cm at 600 rpm was 0.0223 Nm while torque generated by engine was merely 0.004 Nm. So, frictional power losses were more than five times the practical power possible that's why engine was not working.

Final problem with this engine is that tandem arrangement resulting in a very high length of engine so scaling of design would give problem of very great size. for extent ten times engine would be 2.4 meter high which would be very cumbersome design.

Summing up therefore we would say that this engine whilst it has helped us to get a better idea of the Stirling engine, it has severe limitation in term of size accuracy of manufacture and poor thermal separation of the hot and the cold end and result in a crank gear mechanics which is unnecessarily complex.



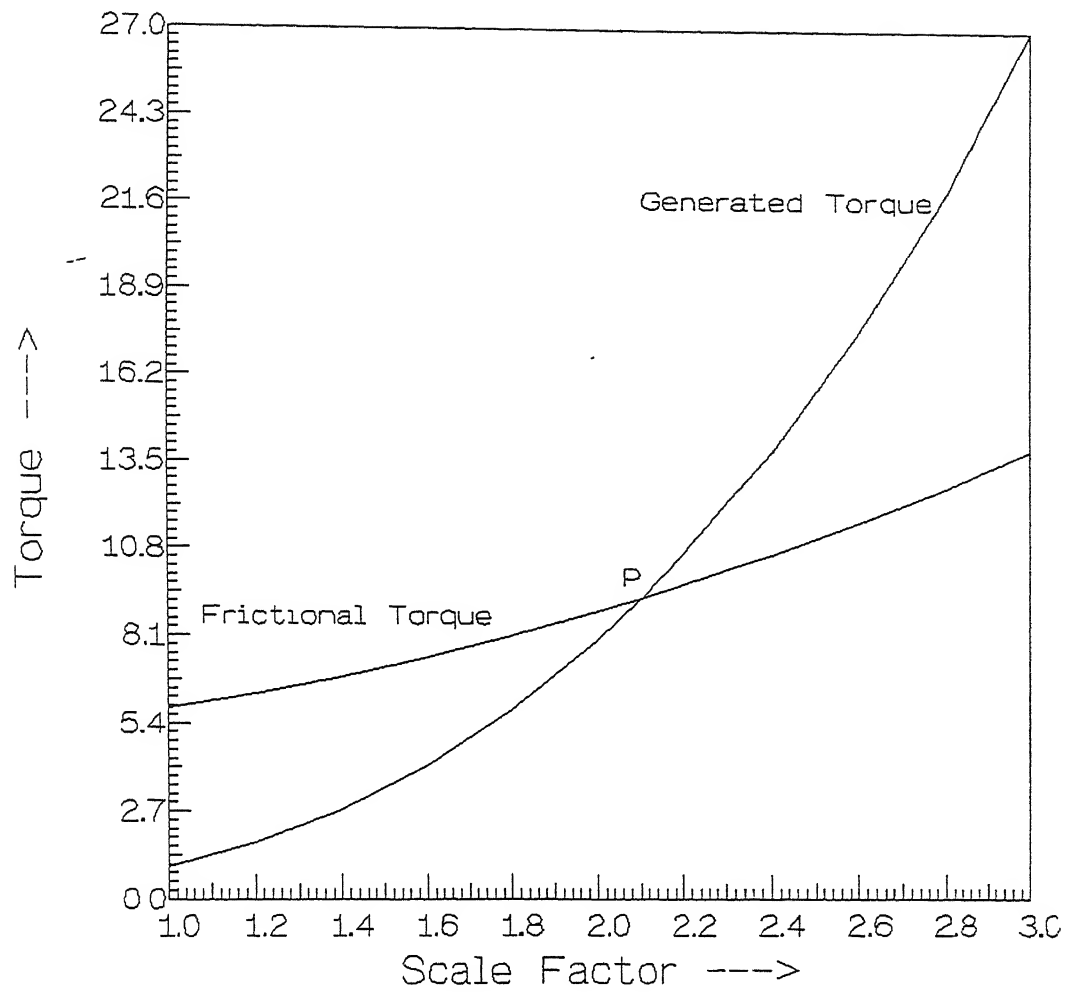


Figure 4 15: Graph showing the plot of frictional power and brake power against swept volume, after point P brake power takes over frictional power and region beyond it gives feasible design

## Chapter 5

# Investigation into $\gamma$ - Stirling Engine

### 5.1 Objectives of the New Design

The proposed design and analysis of  $\gamma$ -Stirling research engine was aimed to fulfill the following objectives :

- To develop a bigger sized engine so that power will be more than frictional forces, as the power produced by Stirling engine varies with the cube of the linear dimension whereas the frictional losses proportional to the square of the linear dimension, so that friction absorbs a lesser proportion of the power produced
- To design a compact and reliable engine which can be manufactured in India without requiring special machining.
- Engine design should be modular so that any type of change in any component can be made without requiring a total rebuild of engine i.e. bore or stroke of the power or the displacer piston can be changed without affecting any of the other part, regenerator or cooler can be modified without too much difficulties. Also, different working fluid can be tested. Similarly, research on different types of seals, dry lubrication and combustion technology can be carried out by taking the advantage of modular design of the engine.

## 5.2 Visualization of Design

As noted earlier, tandem arrangement of the cylinder results in a slender structure which is prone to deflection and jamming of the linkage as was shown by the model and it also results in the heat from hot end convecting to the cold end of the engine. Finally, the very long cross head and seal joint makes it very difficult for this engine to function satisfactorily. From these observation, it was logical development of the design to separate out two cylinder not just configurationally but also spatially. Therefore, engine design was developed as central pedestral with two flywheels and two cranks at  $90^\circ$  and each as cantilever. The air cooling of power side replaced by liquid cooling using water as coolant so that it is possible to vary the coolant temperature from  $100^\circ\text{C}$  to  $0^\circ\text{C}$  and also it will also be easier to measure coolant inlet and outlet temperature accurately. For the hot end temperature, instead of a surface convector type heater, tubular coil surface of the flash steam boiler type is being proposed.

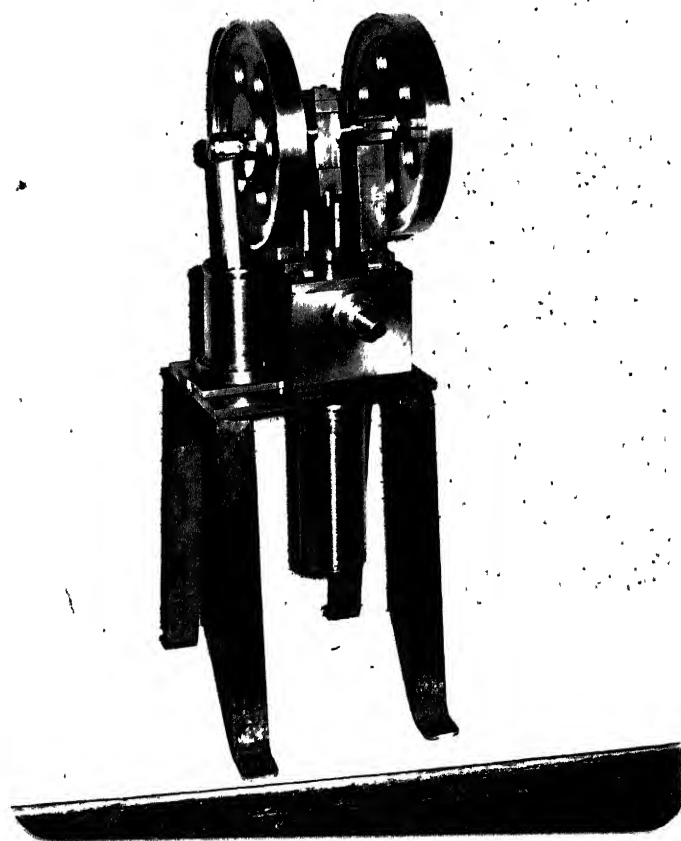
Our objective is not to present an efficient Stirling engine but to have an effective and reliable tool for further research on this subject which will undertake research into the effects on various parameters.

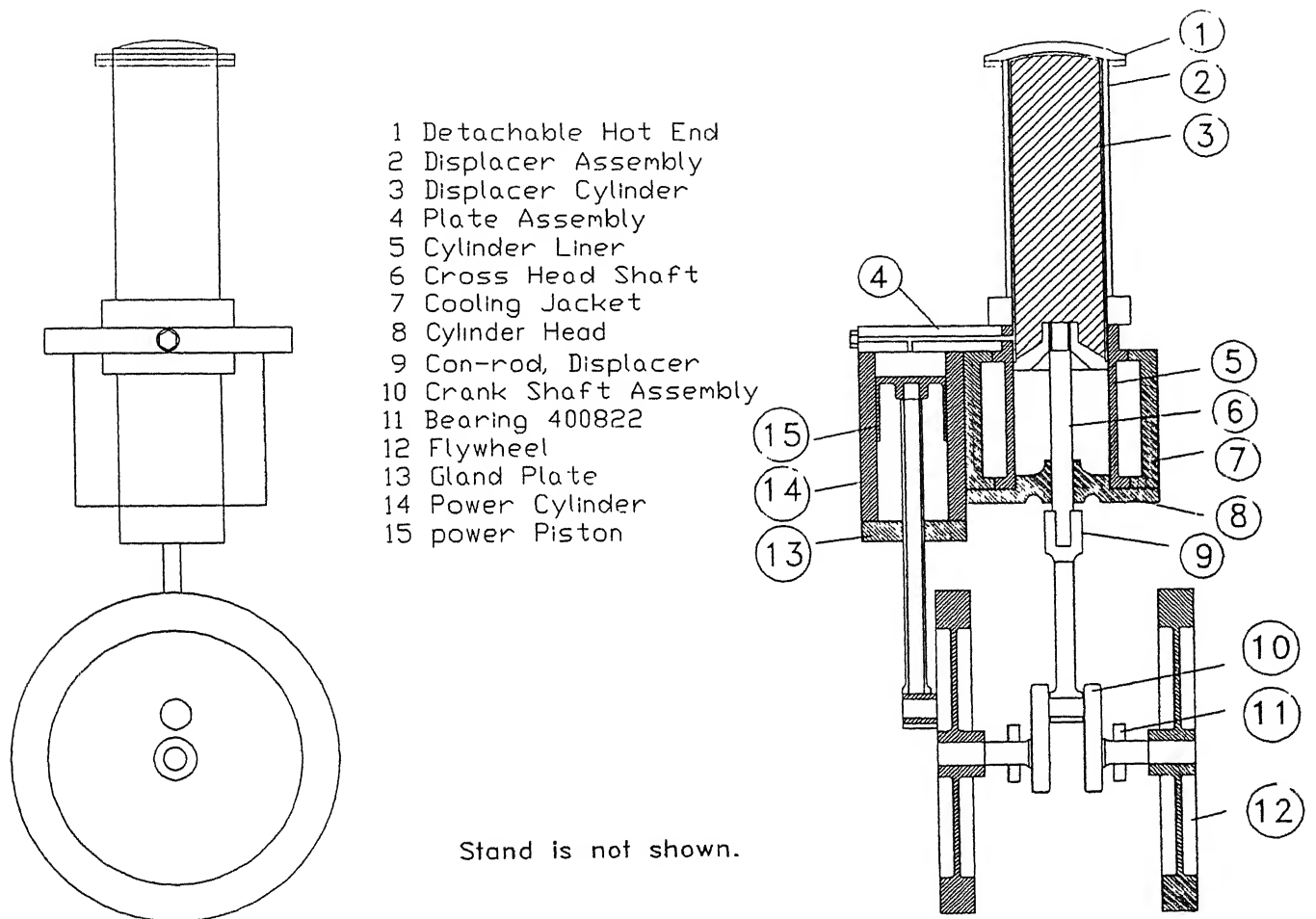
## 5.3 $\gamma$ - Stirling Engine Parameters

Table 5.1 presents the dimensions and performance parameters of the proposed  $\gamma$ - Stirling engine.

Table 5.2 presents the Optimum design parameters(Refer Fig : 3.1) and performance parameters of the proposed  $\gamma$ - Stirling engine.

So, by using consolidated charts, proposed design of  $\gamma$ - Stirling engine has been optimized. There is increase of power output and efficiency improvement over the previous design.



Figure 5 1: Proposed  $\gamma$ - Stirling engine.

Item	Parameter	Value	Reference
Expansion space	Stroke	41 mm	
	Bore	40 mm	
	Swept volume	51.52 cc	
	Temperature	1000.0 K	
Compression space	Stroke	45 mm	
	Bore	30 mm	
	Swept volume	31.8 cc	
	Temperature	300.0 K	
Dead space	Volume	71.93 cc	
Design parameters	$\xi$	.3	
	X	1.44	
	k	.62	
	$\alpha$	90°	
Pressures	Mean pressure	10.94 bar	Eq : 3.13
	Maximum pressure	15.0 bar	Eq : 3.11
	Pressure ratio	1.88	
Speed		1500rpm	
Engine performance	Heat input	616 W	Eq : 3.17
	Power(Schmidt)	431.0 W	Eq : 3.20
	Power(West)	238.0 W	Eq : 3.40
	$\eta$	38.66	

Table 5.1: Design and performance parameters of proposed  $\gamma$ -Stirling engine.

Item	Parameter	Value	Reference
Expansion space	Stroke	41 mm	
	Bore	40 mm	
	Swept volume	51.52 cc	
	Temperature	1000.0 K	
Compression space	Stroke	61.5 mm	
	Bore	30 mm	
	Swept volume	43.73 cc	
	Temperature	300.0 K	
Dead space	Volume	71.93 cc	
Design parameters	$\xi$	.3	
	X	1.44	
	k	.86	
	$\alpha$	97.2°	
Pressures	Mean pressure	10.4 bar	Eq : 3.13
	Maximum pressure	15.0 bar	Eq : 3.11
	Pressure ratio	2.08	
Speed		1500rpm	
Engine performance	Heat input	733 W	Eq : 3.17
	Power(Schmidt)	513.0 W	Eq : 3.20
	Power(West)	288.5 W	Eq : 3.40
	$\eta$	39.35	

Table 5.2: Optimum design and performance parameters of proposed  $\gamma$ -Stirling engine.

## Chapter 6

### Results and Conclusions

As it has been already noted that fabricated model of the engine failed to work. The failure of the engine was attributed due to short power output as compared to the internal friction of engine by a factor of five times. To further investigate the cause of excessive high friction, the engine was stripped, the power piston and power cylinder were checked for clearance and surface finish. The surface finish was measured  $1.6 \mu$  and the clearance was measured to be of the order of .02mm using standard instrument including profilometer. From this it is clear that at this level of workshop practice, the surface finish and friction will also be high. By the same token, leakage as a fraction of the swept volume will be high, hence there is a need of a bigger size engine for any practical purpose. By increasing engine size the effect of leakage as a function of workmanship will be minimized.

It would have been ideal if circularity and circular roughness measuring instrument such as Talyrond was available to measure the circularity of both the cylinder and power piston. This would have checked if leakage past the profile error or jamming between the profile of the piston and cylinder was cause of the error.

In view the proneness of the engine to have high internal friction it would be an good idea to motor the basic engine for several hours before it on test. This would help to get rid of small residual friction and high spots which are inevitable in an assembly. With particular reference to Meede's engine and  $\beta$  configuration in general it would appear from our present experience that this configuration is particularly prone to high friction



and leakage. High friction and leakage are unacceptable in any worthwhile Stirling engine. The sources of these leakages and friction are The long connecting rod between displacer and crank which passes through the power piston. This results in:

- Two additional leakage path through the power piston which is bad feature,
- The long connecting rod itself may bow and bend since it passes through two sealing point there is chance of unacceptable level of friction

From this it would be our advice and conclusion that future research into designing and building Stirling engine should tend to avoid the  $\beta$ - configuration unless there are acceptable and compelling reason for choosing this particular configuration. It is true that in  $\beta$ - engine there is less dead volume ratio which results in higher efficiency. This temptation must be firmly avoided at the beginning of the Stirling engine programme where it is important to generate a set of baseline data and technology so that future design capacity established. The  $\gamma$ - engine configuration is thus preferred.

Friction being an such a important parameter that following actions are strongly recommended:

- All mating surface and in particular the contact between power cylinder and power piston should be lapped or honed.
- Plain bearing should be replaced by rolling bearing as soon the scale of components permits.
- Whilst any western design may give a certain flywheel size it may be wiser to replace flywheel design with a heavier flywheel because western design presume standard of fits and manufacture which may not exist in the institute workshop.

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# Appendix

## Working Fluids

Air, hydrogen and helium can be used as working fluids in Stirling engine . Air has many advantages. It is abundant. It is easier to seal than the light gases and leakages can be tolerated for the air escaping has no adverse environment impact. But for high specific output competing with the internal combustion gasoline or diesel engines, Stirling engines must operate at high speed with very high pressure levels and have to use a light gas working fluid such as hydrogen and helium.

A major problem in the use of light gas working fluid is the difficulty of their containment. Both helium, and hydrogen are extremely mobile, and very difficult to seal effectively at high pressure over the long term. Hydrogen has wider flammability limits than any other element in oxygen or air so leakages in confined spaces can be hazardous. Helium is inert and on grounds of safety is much preferred.